Electricity and Magnetism
Electric Field

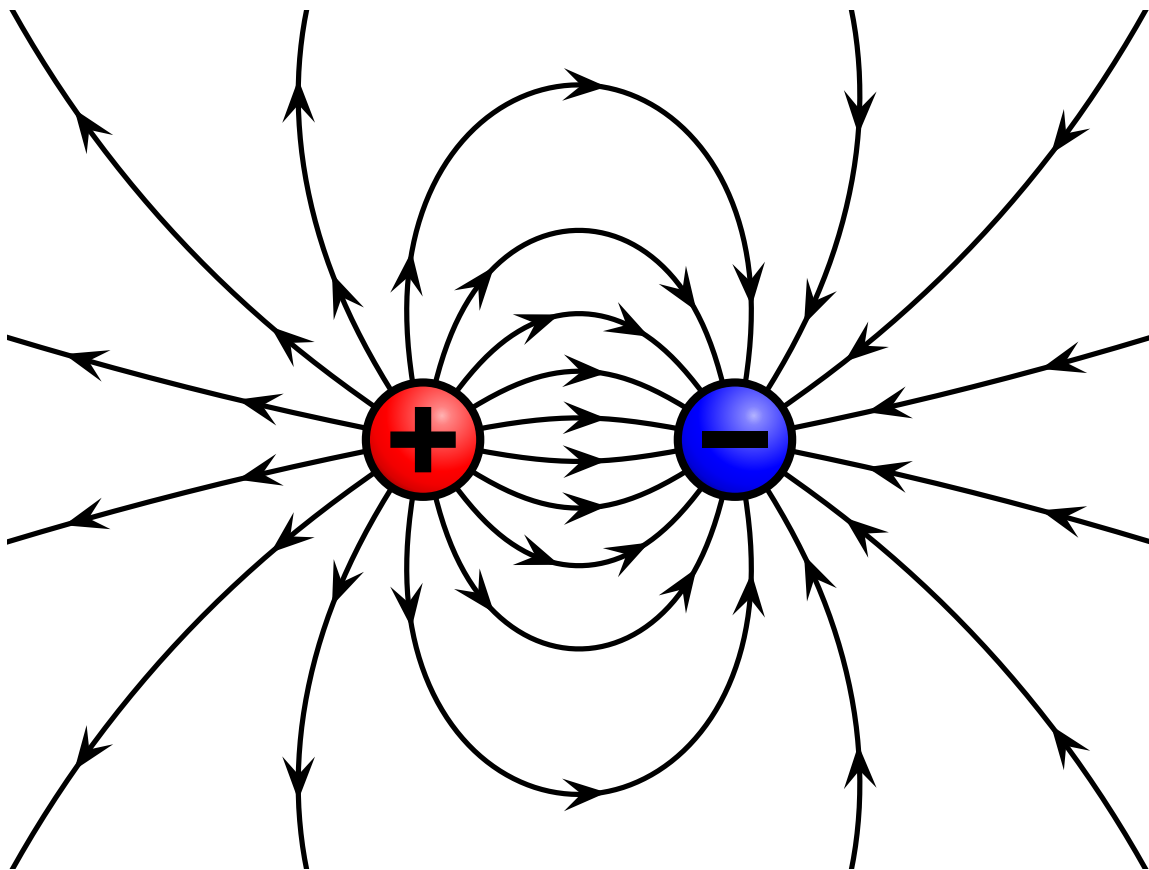# Contents

Chapter 1

Electric charge

Electric field of a positive and a negative point charge.

Electric charge is the physical property of matter that causes it to experience a force when placed in an electromagnetic field. There are two types of electric charges: positive and negative. Positively charged substances are repelled from other positively charged substances, but attracted to negatively charged substances; negatively charged substances are repelled from negative and attracted to positive. An object is negatively charged if it has an excess of electrons, and is otherwise positively charged or uncharged. The SI derived unit of electric charge is the coulomb (C), although in electrical engineering it is also common to use the ampere-hour (Ah), and in chemistry it is common to use the elementary charge (e) as a unit. The symbol Q is often used to denote charge. The early knowledge of how charged substances interact is now called classical electrodynamics, and is still very accurate if quantum effects do not need to be considered.

The *electric charge* is a fundamental conserved property of some subatomic particles, which determines their electromagnetic interaction. Electrically charged matter is influenced by, and produces, electromagnetic fields. The interaction be-

tween a moving charge and an electromagnetic field is the source of the electromagnetic force, which is one of the four fundamental forces (See also: magnetic field).

Twentieth-century experiments demonstrated that electric charge is *quantized*; that is, it comes in integer multiples of individual small units called the elementary charge, *e*, approximately equal to 1.602×10^{-19} coulombs (except for particles called quarks, which have charges that are integer multiples of *e/3*). The proton has a charge of +*e*, and the electron has a charge of −*e*. The study of charged particles, and how their interactions are mediated by photons, is called quantum electrodynamics.

1.1   Overview
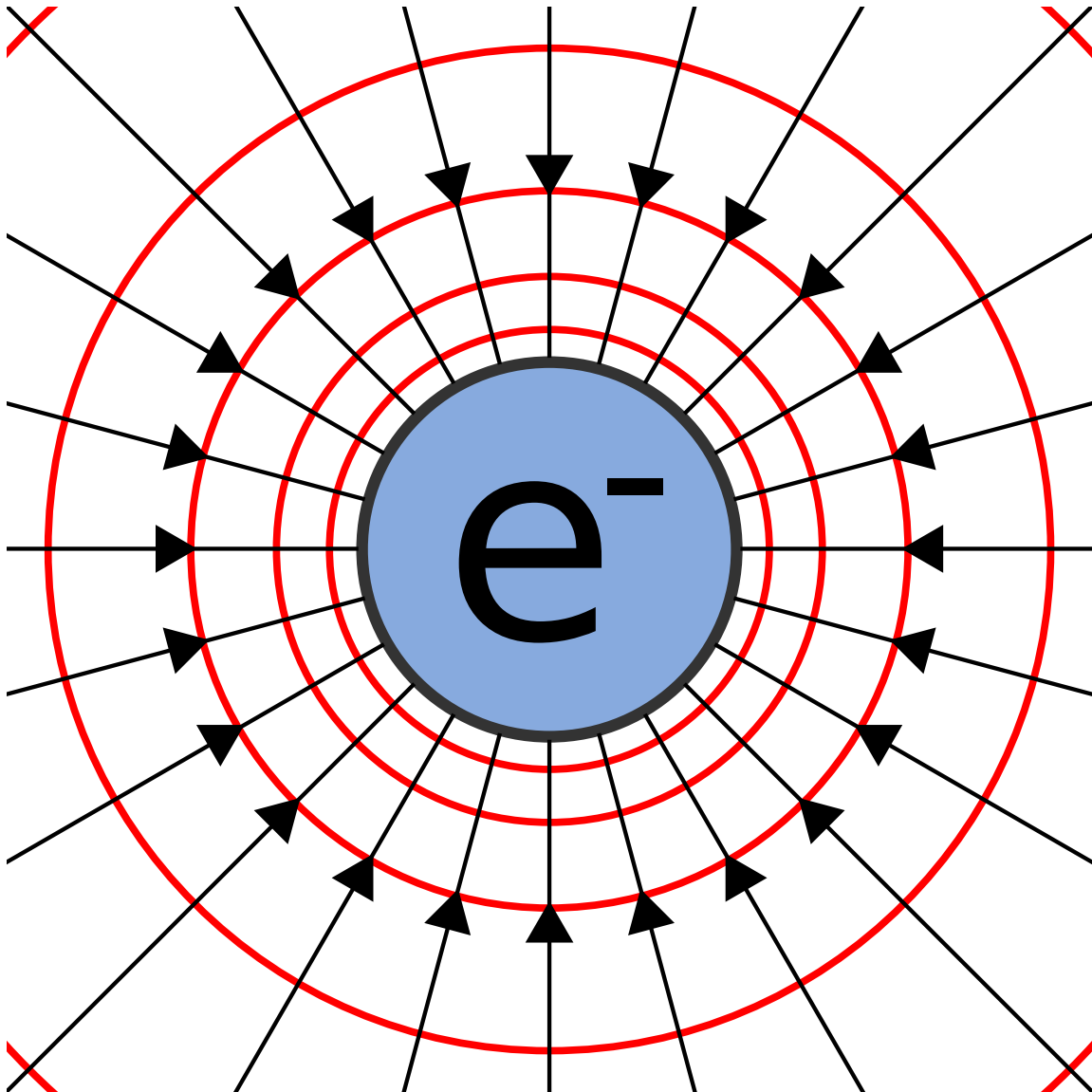
Diagram showing field lines and equipotentials around an electron, a negatively charged particle. In an electrically neutral atom, the number of electrons is equal to the number of protons (which are positively charged), resulting in a net zero overall charge

Charge is the fundamental property of forms of matter that exhibit electrostatic attraction or repulsion in the presence of other matter. Electric charge is a characteristic property of many subatomic particles. The charges of free-standing particles are integer multiples of the elementary charge *e*; we say that electric charge is *quantized*. Michael Faraday, in his electrolysis experiments, was the first to note the discrete nature of electric charge. Robert Millikan's oil-drop experiment demonstrated this fact directly, and measured the elementary charge.

By convention, the charge of an electron is −1, while that of a proton is +1. Charged particles whose charges have

the same sign repel one another, and particles whose charges have different signs attract. Coulomb's law quantifies the electrostatic force between two particles by asserting that the force is proportional to the product of their charges, and inversely proportional to the square of the distance between them.

The charge of an antiparticle equals that of the corresponding particle, but with opposite sign. Quarks have fractional charges of either $-\frac{1}{3}$ or $+\frac{2}{3}$, but free-standing quarks have never been observed (the theoretical reason for this fact is asymptotic freedom).

The electric charge of a macroscopic object is the sum of the electric charges of the particles that make it up. This charge is often small, because matter is made of atoms, and atoms typically have equal numbers of protons and electrons, in which case their charges cancel out, yielding a net charge of zero, thus making the atom neutral.

An *ion* is an atom (or group of atoms) that has lost one or more electrons, giving it a net positive charge (cation), or that has gain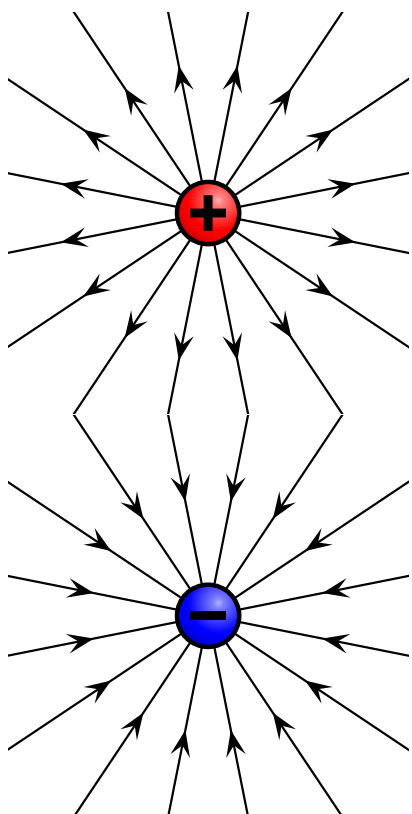ed one or more electrons, giving it a net negative charge (anion). *Monatomic ions* are formed from single atoms, while *polyatomic ions* are formed from two or more atoms that have been bonded together, in each case yielding an ion with a positive or negative net charge.

Electric field induced by a positive electric charge (left) and a field induced by a negative electric charge (right).

During formation of macroscopic objects, constituent atoms and ions usually combine to form structures composed of neutral *ionic compounds* electrically bound to neutral atoms. Thus macroscopic objects tend toward being neutral overall, but macroscopic objects are rarely perfectly net neutral.

Sometimes macroscopic objects contain ions distributed throughout the material, rigidly bound in place, giving an overall net positive or negative charge to the object. Also, macroscopic objects made of conductive elements, can more or less easily (depending on the element) take on or give off electrons, and then maintain a net negative or positive charge indefinitely. When the net electric charge of an object is non-zero and motionless, the phenomenon is known as static electricity. This can easily be produced by rubbing two dissimilar materials together, such as rubbing amber with fur or glass with silk. In this way non-conductive materials can be charged to a significant degree, either positively or negatively. Charge taken from one material is moved to the other material, leaving an opposite charge of the same magnitude behind. The law of *conservation of charge* always applies, giving the object from which a negative charge has been taken a positive charge of the same magnitude, and vice versa.

Even when an object's net charge is zero, charge can be distributed non-uniformly in the object (e.g., due to an external electromagnetic field, or bound polar molecules). In such cases the object is said to be polarized. The charge due to polarization is known as bound charge, while charge on an object produced by electrons gained or lost from

outside the object is called *free charge*. The motion of electrons in conductive metals in a specific direction is known as electric current.

1.2   Units

The SI unit of quantity of electric charge is the coulomb, which is equivalent to about 6.242×10^{18} *e* (*e* is the charge of a proton). Hence, the charge of an electron is approximately -1.602×10^{-19} C. The coulomb is defined as the quantity of charge that has passed through the cross section of an electrical conductor carrying one ampere within one second. The symbol *Q* is often used to denote a quantity of electricity or charge. The quantity of electric charge can be directly measured with an electrometer, or indirectly measured with a ballistic galvanometer.

After finding the quantized character of charge, in 1891 George Stoney proposed the unit 'electron' for this fundamental unit of electrical charge. This was before the discovery of the particle by J.J. Thomson in 1897. The unit is today treated as nameless, referred to as "elementary charge", "fundamental unit of charge", or simply as "e". A measure of charge should be a multiple of the elementary charge *e*, even if at large scales charge seems to behave as a real quantity. In some contexts it is meaningful to speak of fractions of a charge; for example in the charging of a capacitor, or in the fractional quantum Hall effect.

In systems of units other than SI such as cgs, electric charge is expressed as combination of only three fundamental quantities such as length, mass and time and not four as in SI where electric charge is a combination of length, mass, time and electric current.

1.3   History

As reported by the ancient Greek mathematician Thales of Miletus around 600 BC, charge (or *electricity*) could be accumulated by rubbing fur on various substances, such as amber. The Greeks noted that the charged amber buttons could attract light objects such as hair. They also noted that if they rubbed the amber for long enough, they could even get an electric spark to jump. This property derives from the triboelectric effect.

In 1600, the English scientist William Gilbert returned to the subject in *De Magnete*, and coined the New Latin word *electricus* from ηλεκτρον (*elektron*), the Greek word for *amber*, which soon gave rise to the English words "electric" and "electricity." He was followed in 1660 by Otto von Guericke, who invented what was probably the first electrostatic generator. Other European pioneers were Robert Boyle, who in 1675 stated that electric attraction and repulsion can act across a vacuum; Stephen Gray, who in 1729 classified materials as conductors and insulators; and C. F. du Fay, who proposed in 1733[1] that electricity comes in two varieties that cancel each other, and expressed this in terms of a two-fluid theory. When glass was rubbed with silk, du Fay said that the glass was charged with *vitreous electricity*, and, when amber was rubbed with fur, the amber was said to be charged with *resinous electricity*. In 1839, Michael Faraday showed that the apparent division between static electricity, current electricity, and bioelectricity was incorrect, and all were a consequence of the behavior of a single kind of electricity appearing in opposite polarities. It is arbitrary which polarity is called positive and which is called negative. Positive charge can be defined as the charge left on a glass rod after being rubbed with silk.[2]

One of the foremost experts on electricity in the 18th century was Benjamin Franklin, who argued in favour of a one-fluid theory of electricity. Franklin imagined electricity as being a type of invisible fluid present in all matter; for example, he believed that it was the glass in a Leyden jar that held the accumulated charge. He posited that rubbing insulating surfaces together caused this fluid to change location, and that a flow of this fluid constitutes an electric current. He also posited that when matter contained too little of the fluid it was "negatively" charged, and when it had an excess it was "positively" charged. For a reason that was not recorded, he identified the term "positive" with vitreous electricity and "negative" with resinous electricity. William Watson arrived at the same explanation at about the same time.

1.4   Static electricity and electric current

Static electricity and electric current are two separate phenomena. They both involve electric charge, and may occur simultaneously in the same object. Static electricity refers to the electric charge of an object and the related electrostatic discharge when two objects are brought together that are not at equilibrium. An electrostatic discharge
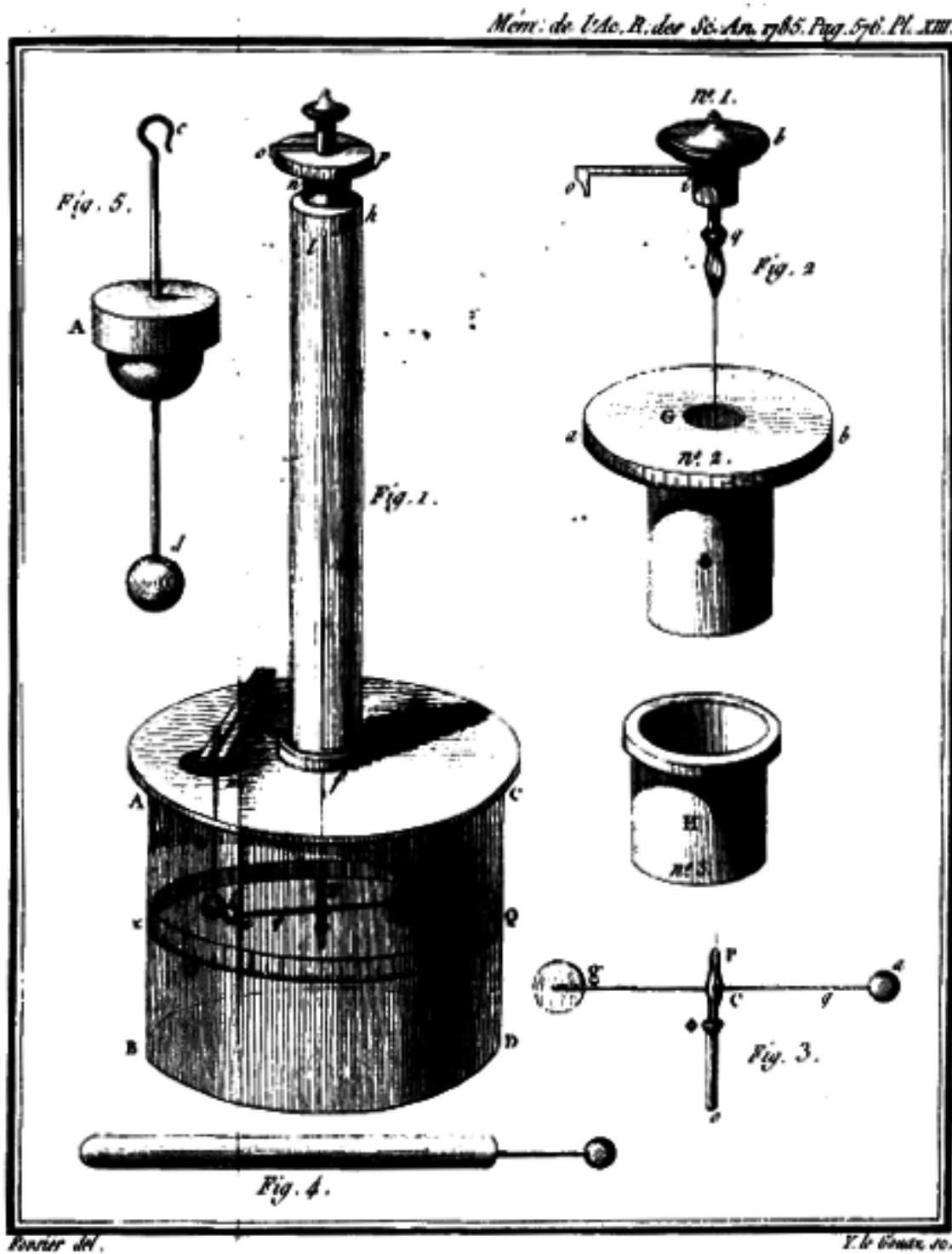
Coulomb's torsion balance

creates a change in the charge of each of the two objects. In contrast, electric current is the flow of electric charge through an object, which produces no net loss or gain of electric charge.

1.4.1 Electrification by friction

Further information: triboelectric effect

When a piece of glass and a piece of resin—neither of which exhibit any electrical properties—are rubbed together and left with the rubbed surfaces in contact, they still exhibit no electrical properties. When separated, they attract each other.

A second piece of glass rubbed with a second piece of resin, then separated and suspended near the former pieces of glass and resin causes these phenomena:

- The two pieces of glass repel each other.

- Each piece of glass attracts each piece of resin.

- The two pieces of resin repel each other.

This attraction and repulsion is an *electrical phenomena,* and the bodies that exhibit them are said to be *electrified*, or *electrically charged*. Bodies may be electrified in many other ways, as well as by friction. The electrical properties of the two pieces of glass are similar to each other but opposite to those of the two pieces of resin: The glass attracts what the resin repels and repels what the resin attracts.

If a body electrified in any manner whatsoever behaves as the glass does, that is, if it repels the glass and attracts the resin, the body is said to be 'vitreously' electrified, and if it attracts the glass and repels the resin it is said to be 'resinously' electrified. All electrified bodies are found to be either vitreously or resinously electrified.

It is the established convention of the scientific community to define the vitreous electrification as positive, and the resinous electrification as negative. The exactly opposite properties of the two kinds of electrification justify our indicating them by opposite signs, but the application of the positive sign to one rather than to the other kind must be considered as a matter of arbitrary convention, just as it is a matter of convention in mathematical diagram to reckon positive distances towards the right hand.

No force, either of attraction or of repulsion, can be observed between an electrified body and a body not electrified.[3]

Actually, all bodies are electrified, but may appear not to be so by the relative similar charge of neighboring objects in the environment. An object further electrified + or – creates an equivalent or opposite charge by default in neighboring objects, until those charges can equalize. The effects of attraction can be observed in high-voltage experiments, while lower voltage effects are merely weaker and therefore less obvious. The attraction and repulsion forces are codified by Coulomb's Law (attraction falls off at the square of the distance, which has a corollary for acceleration in a gravitational field, suggesting that gravitation may be merely electrostatic phenomenon between relatively weak charges in terms of scale). See also the Casimir effect.

It is now known that the Franklin/Watson model was fundamentally correct. There is only one kind of electrical charge, and only one variable is required to keep track of the amount of charge.[4] On the other hand, just knowing the charge is not a complete description of the situation. Matter is composed of several kinds of electrically charged particles, and these particles have many properties, not just charge.

The most common charge carriers are the positively charged proton and the negatively charged electron. The movement of any of these charged particles constitutes an electric current. In many situations, it suffices to speak of the *conventional current* without regard to whether it is carried by positive charges moving in the direction of the conventional current or by negative charges moving in the opposite direction. This macroscopic viewpoint is an approximation that simplifies electromagnetic concepts and calculations.

At the opposite extreme, if one looks at the microscopic situation, one sees there are many ways of carrying an electric current, including: a flow of electrons; a flow of electron "holes" that act like positive particles; and both negative and positive particles (ions or other charged particles) flowing in opposite directions in an electrolytic solution or a plasma.

Beware that, in the common and important case of metallic wires, the direction of the conventional current is opposite to the drift velocity of the actual charge carriers, i.e., the electrons. This is a source of confusion for beginners.

1.5   Properties

Aside from the properties described in articles about electromagnetism, charge is a relativistic invariant. This means that any particle that has charge Q, no matter how fast it goes, always has charge Q. This property has been experimentally verified by showing that the charge of *one* helium nucleus (two protons and two neutrons bound together in

a nucleus and moving around at high speeds) is the same as *two* deuterium nuclei (one proton and one neutron bound together, but moving much more slowly than they would if they were in a helium nucleus).

1.6 Conservation of electric charge

Main article: Charge conservation

The total electric charge of an isolated system remains constant regardless of changes within the system itself. This law is inherent to all processes known to physics and can be derived in a local form from gauge invariance of the wave function. The conservation of charge results in the charge-current continuity equation. More generally, the net change in charge density ϱ within a volume of integration V is equal to the area integral over the current density $\mathbf{J}$ through the closed surface $S = \partial V$, which is in turn equal to the net current I:

$$-\frac{d}{dt} \int_V \rho \, \mathrm{d}V = \oiint_{\partial V} \mathbf{J} \cdot \mathrm{d}\mathbf{S} = \int J dS \cos\theta = I.$$

Thus, the conservation of electric charge, as expressed by the continuity equation, gives the result:

$$I = \frac{dQ}{dt}.$$

The charge transferred between times t_i and t_f is obtained by integrating both sides:

$$Q = \int_{t_i}^{t_f} I \, \mathrm{d}t$$

where I is the net outward current through a closed surface and Q is the electric charge contained within the volume defined by the surface.

1.7 See also

- Quantity of electricity
- SI electromagnetism units

1.8 References

[1] Two Kinds of Electrical Fluid: Vitreous and Resinous – 1733

[2] Electromagnetic Fields (2nd Edition), Roald K. Wangsness, Wiley, 1986. ISBN 0-471-81186-6 (intermediate level textbook)

[3] James Clerk Maxwell *A Treatise on Electricity and Magnetism*, pp. 32-33, Dover Publications Inc., 1954 ASIN: B000HFDK0K, 3rd ed. of 1891

[4] One Kind of Charge

1.9 External links

- How fast does a charge decay?
- Science Aid: Electrostatic charge Easy-to-understand page on electrostatic charge.
- History of the electrical units.

Chapter 2

Electrostatic induction

Electrostatic induction is a redistribution of electrical charge in an object, caused by the influence of nearby charges.[1] In the presence of a charged body, an insulated conductor develops a positive charge on one end and a negative charge on the other end.[1] Induction was discovered by British scientist John Canton in 1753 and Swedish professor Johan Carl Wilcke in 1762.[2] Electrostatic generators, such as the Wimshurst machine, the Van de Graaff generator and the electrophorus, use this principle. Due to induction, the electrostatic potential (voltage) is constant at any point throughout a conductor.[3] Induction is also responsible for the attraction of light nonconductive objects, such as balloons, paper or styrofoam scraps, to static electric charges. Electrostatic induction should not be confused with electromagnetic induction.

2.1 Explanation
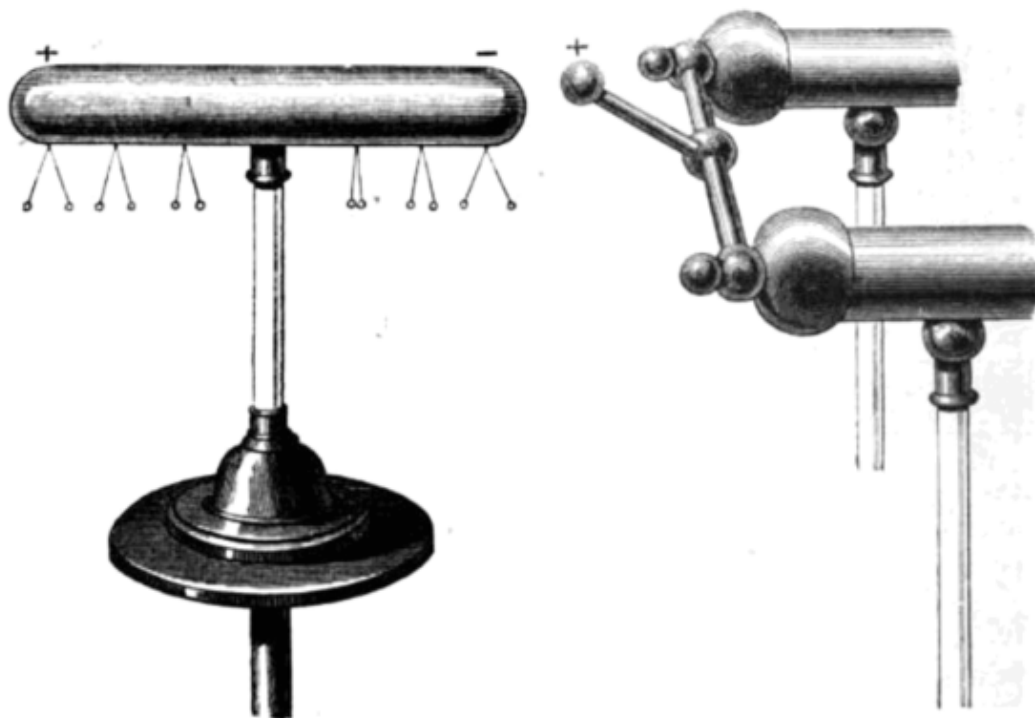
Demonstration of induction, in the 1870s. The positive terminal of an electrostatic machine is placed near an uncharged brass cylinder, causing the left end to acquire a positive charge and the right to acquire a negative charge. The small pith ball electroscopes hanging from the bottom show that the charge is concentrated at the ends.

A normal uncharged piece of matter has equal numbers of positive and negative electric charges in each part of it, located close together, so no part of it has a net electric charge. The positive charges are the atoms' nuclei which are bound into the structure of matter and are not free to move. The negative charges are the atoms' electrons. In electrically conductive objects such as metals, some of the electrons are able to move freely about in the object.

When a charged object is brought near an uncharged, electrically conducting object, such as a piece of metal, the force of the nearby charge due to Coulomb's law causes a separation of these internal charges. For example, if a positive charge is brought near the object (see picture at right), the electrons in the metal will be attracted toward it and move to the side of the object facing it. When the electrons move out of an area, they leave an unbalanced positive charge due to the nuclei. This results in a region of negative charge on the object nearest to the external charge, and a region of positive charge on the part away from it. These are called *induced charges*. If the external charge is negative, the polarity of the charged regions will be reversed.

Since this process is just a redistribution of the charges that were already in the object, it doesn't change the *total* charge on the object; it still has no net charge. This induction effect is reversible; if the nearby charge is removed, the attraction between the positive and negative internal charges cause them to intermingle again.

2.2 Charging an object by induction
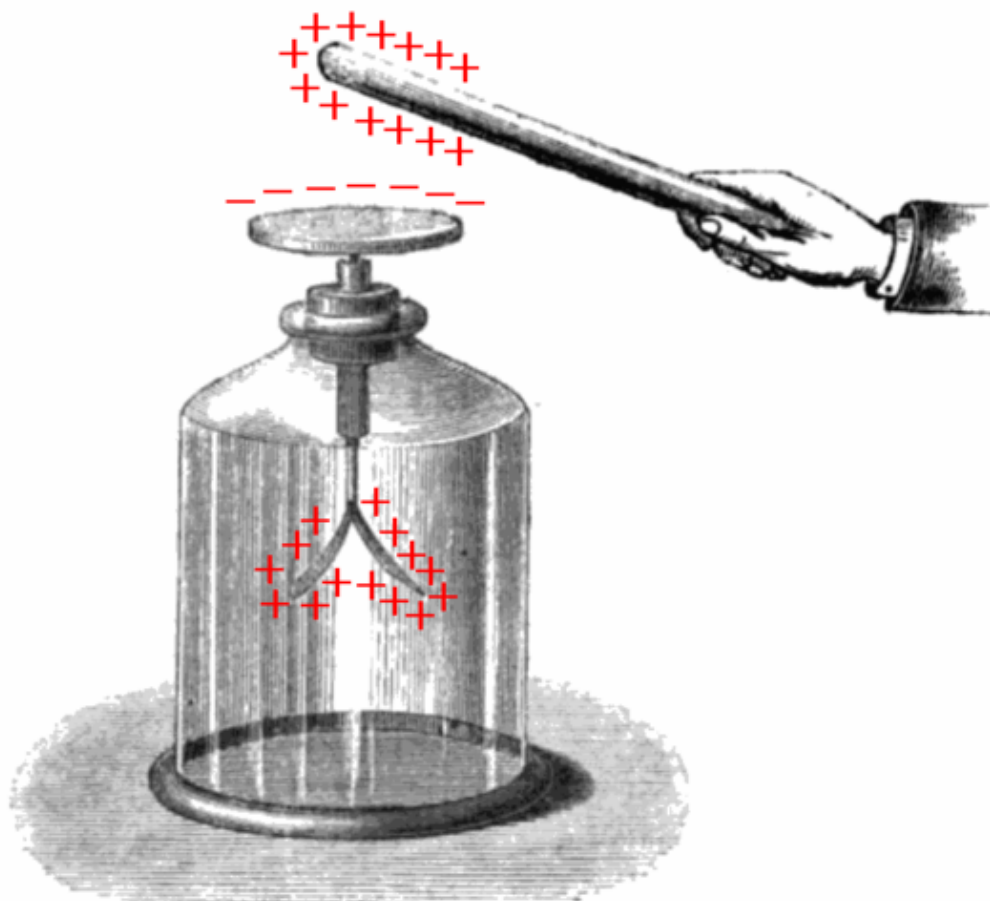
Gold-leaf electroscope, showing induction, before the terminal is grounded.

However, the induction effect can also be used to put a net charge on an object. If, while it is close to the positive charge, the above object is momentarily connected through a conductive path to electrical ground, which is a large reservoir of both positive and negative charges, some of the negative charges in the ground will flow into the object, under the attraction of the nearby positive charge. When the contact with ground is broken, the object is left with a net negative charge.

This method can be demonstrated using a gold-leaf electroscope, which is an instrument for detecting electric charge. The electroscope is first discharged, and a charged object is then brought close to the instrument's top terminal. Induction causes a separation of the charges inside the electroscope's metal rod, so that the top terminal gains a net charge of opposite polarity to that of the object, while the gold leaves gain a charge of the same polarity. Since both leaves have the same charge, they repel each other and spread apart. The electroscope has not acquired a net charge: the charge within it has merely been redistributed, so if the charged object were to be moved away from the electroscope the leaves will come together again.

But if an electrical contact is now briefly made between the electroscope terminal and ground, for example by touching the terminal with a finger, this causes charge to flow from ground to the terminal, attracted by the charge on the object close to the terminal. This charge neutralizes the charge in the gold leaves, so the leaves come together again. The electroscope now contains a net charge opposite in polarity to that of the charged object. When the electrical contact to earth is broken, e.g. by lifting the finger, the extra charge that has just flowed into the electroscope cannot escape, and the instrument retains a net charge. The charge is held in the top of the electroscope terminal by the attraction of the inducing charge. But when the inducing charge is moved away, the charge is released and spreads throughout the electroscope terminal to the leaves, so the gold leaves move apart again.

The sign of the charge left on the electroscope after grounding is always opposite in sign to the external inducing charge.[4] The two rules of induction are:[4][5]

- If the object is not grounded, the nearby charge will induce *equal* and *opposite* charges in the object.

- If *any part* of the object is momentarily grounded while the inducing charge is near, a charge opposite in polarity to the inducing charge will be attracted from ground into the object, and it will be left with a charge *opposite* to the inducing charge.

2.3   The electrostatic field inside a conductive object is zero

A remaining question is how large the induced charges are. The movement of charge is caused by the force exerted by the electric field of the external charged object, by Coulomb's law. As the charges in the metal object continue to separate, the resulting positive and negative regions create their own electric field, which opposes the field of the external charge.[3] This process continues until very quickly (within a fraction of a second) an equilibrium is reached in which the induced charges are exactly the right size to cancel the external electric field throughout the interior of the metal object.[3][6] Then the remaining mobile charges (electrons) in the interior of the metal no longer feel a force and the net motion of the charges stops.[3]

2.4   Induced charge resides on the surface

Since the mobile charges in the interior of a metal object are free to move in any direction, there can never be a static concentration of charge inside the metal; if there was, it would attract opposite polarity charge to neutralize it.[3] Therefore in induction, the mobile charges move under the influence of the external charge until they reach the surface of the metal and collect there, where they are constrained from moving by the boundary.[3]

This establishes the important principle that electrostatic charges on conductive objects reside on the surface of the object.[3][6] External electric fields induce surface charges on metal objects that exactly cancel the field within.[3] Since the field is the gradient of the electrostatic potential, another way of saying this is that in electrostatics, the potential (voltage) throughout a conductive object is constant.[3]

2.5   Induction in dielectric objects

A similar induction effect occurs in nonconductive (dielectric) objects, and is responsible for the attraction of small light nonconductive objects, like balloons, scraps of paper or Styrofoam, to static electric charges.[7][8][9][10] In non-conductors, the electrons are bound to atoms or molecules and are not free to move about the object as in conductors; however they can move a little within the molecules.

If a positive charge is brought near a nonconductive object, the electrons in each molecule are attracted toward it, and move to the side of the molecule facing the charge, while the positive nuclei are repelled and move slightly to
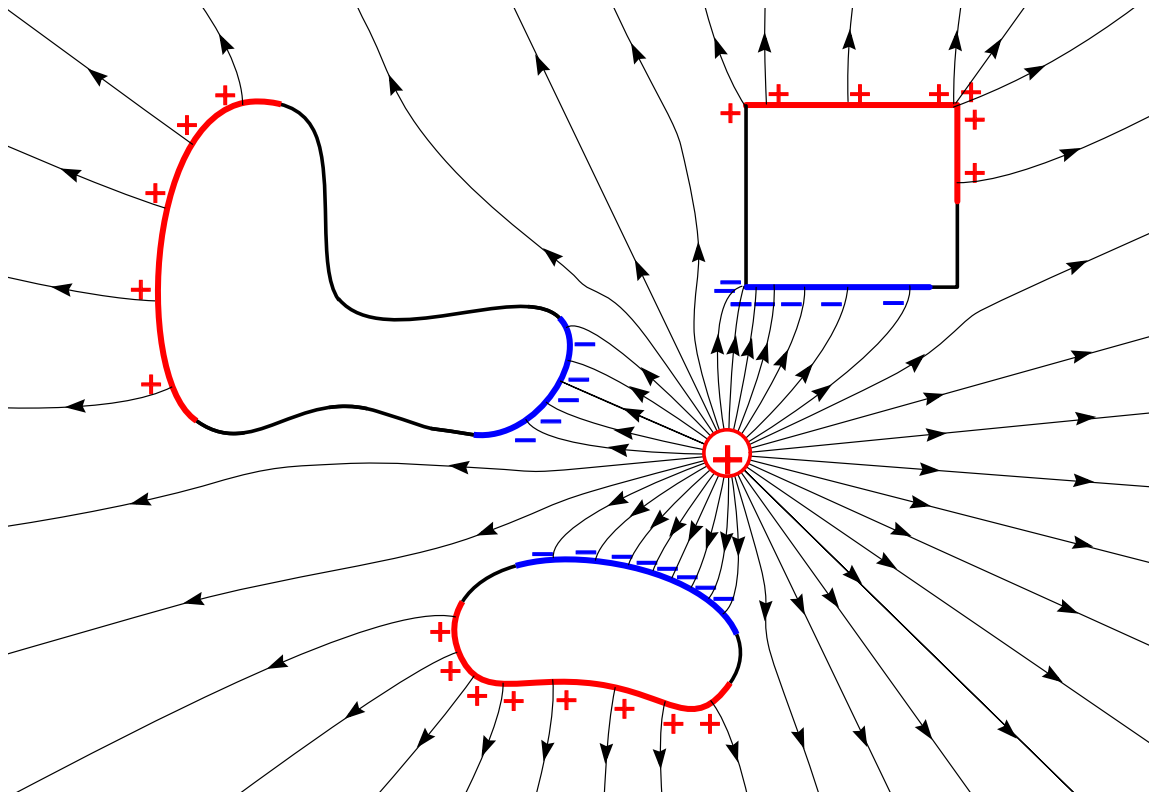
Surface charges induced in metal objects by a nearby charge. The electrostatic field (lines with arrows) of a nearby positive charge (+) causes the mobile charges in metal objects to separate. Negative charges (blue) are attracted and move to the surface of the object facing the external charge. Positive charges (red) are repelled and move to the surface facing away. These induced surface charges create an opposing electric field that exactly cancels the field of the external charge throughout the interior of the metal. Therefore electrostatic induction ensures that the electric field everywhere inside a conductive object is zero.
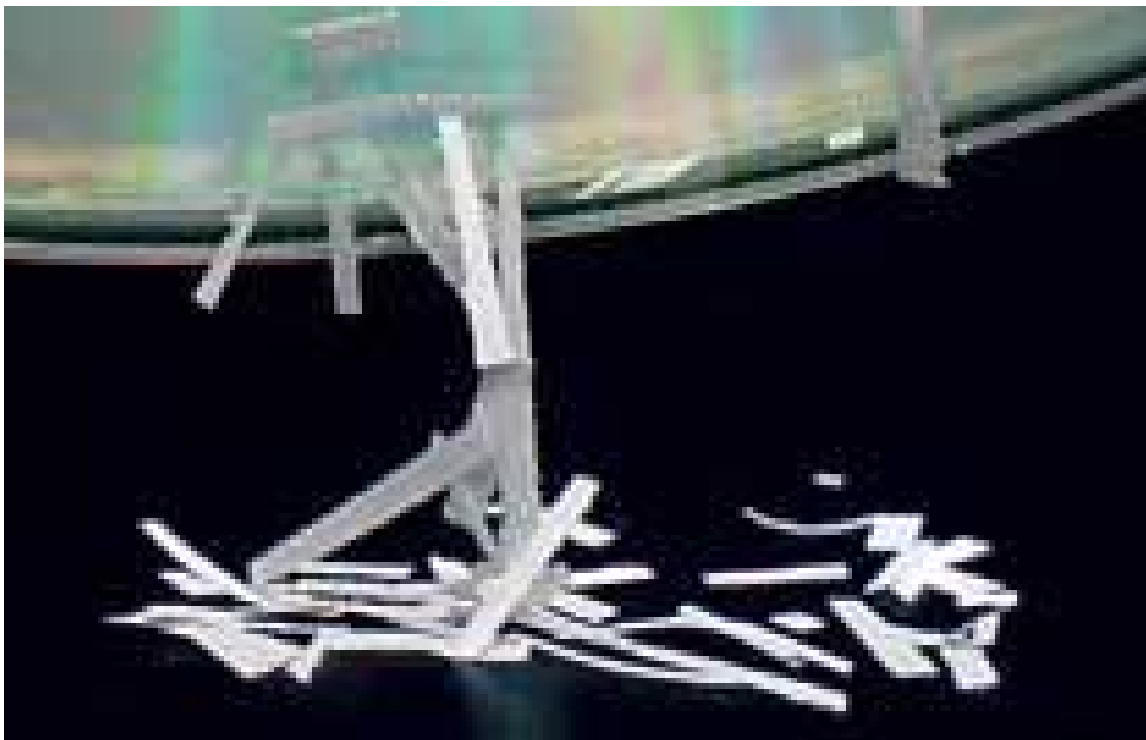
the opposite side of the molecule. Since the negative charges are now closer to the external charge than the positive

charges, their attraction is greater than the repulsion of the positive charges, resulting in a small net attraction of the molecule toward the charge. This is called polarization, and the polarized molecules are called dipoles. This effect is microscopic, but since there are so many molecules, it adds up to enough force to move a light object like Styrofoam. This is the principle of operation of a pith-ball electroscope.[11]

2.6   Notes

[1] "Electrostatic induction". *Encyclopaedia Britannica Online*. Encyclopaedia Britannica, Inc. 2008. Retrieved 2008-06-25.

[2] "Electricity". *Encyclopaedia Britannica, 11th Ed.* **9**. The Encyclopaedia Britannica Co. 1910. p. 181. Retrieved 2008-06-23.

[3] Purcell, Edward M.; David J. Morin (2013). *Electricity and Magnetism*. Cambridge Univ. Press. pp. 127–128. ISBN 1107014026.

[4] Cope, Thomas A. Darlington. *Physics*. Library of Alexandria. ISBN 1465543724.

[5] Hadley, Harry Edwin (1899). *Magnetism & Electricity for Beginners*. Macmillan & Company. p. 182.

[6] Saslow, Wayne M. (2002). *Electricity, magnetism, and light*. US: Academic Press. pp. 159–161. ISBN 0-12-619455-6.

[7] Sherwood, Bruce A.; Ruth W. Chabay (2011). *Matter and Interactions, 3rd Ed.* USA: John Wiley and Sons. pp. 594–596. ISBN 0-470-50347-5.

[8] Paul E. Tippens, *Electric Charge and Electric Force*, Powerpoint presentation, p.27-28, 2009, S. Polytechnic State Univ. on DocStoc.com website

[9] Henderson, Tom (2011). "Charge and Charge Interactions". *Static Electricity, Lesson 1*. The Physics Classroom. Retrieved 2012-01-01.

[10] Winn, Will (2010). *Introduction to Understandable Physics Vol. 3: Electricity, Magnetism and Ligh*. USA: Author House. p. 20.4. ISBN 1-4520-1590-2.

[11] *Kaplan MCAT Physics 2010-2011*. USA: Kaplan Publishing. 2009. p. 329. ISBN 1-4277-9875-3.

2.7   External links

- "Charging by electrostatic induction". *Regents exam prep center*. Oswego City School District. 1999. Retrieved 2008-06-25.

Chapter 3

Coulomb's law

Coulomb's law, or **Coulomb's** inverse-square law, is a law of physics describing the electrostatic interaction between electrically charged particles. The law was first published in 1784 by French physicist Charles Augustin de Coulomb and was essential to the development of the theory of electromagnetism. It is analogous to Isaac Newton's inverse-square law of universal gravitation. Coulomb's law can be used to derive Gauss's law, and vice versa. The law has been tested heavily, and all observations have upheld the law's principle.

3.1   History

Ancient cultures around the Mediterranean knew that certain objects, such as rods of amber, could be rubbed with cat's fur to attract light objects like feathers. Thales of Miletus made a series of observations on static electricity around 600 BC, from which he believed that friction rendered amber magnetic, in contrast to minerals such as magnetite, which needed no rubbing.[1][2] Thales was incorrect in believing the attraction was due to a magnetic effect, but later science would prove a link between magnetism and electricity. Electricity would remain little more than an intellectual curiosity for millennia until 1600, when the English scientist William Gilbert made a careful study of electricity and magnetism, distinguishing the lodestone effect from static electricity produced by rubbing amber.[1] He coined the New Latin word *electricus* ("of amber" or "like amber", from ἤλεκτρον [*elektron*], the Greek word for "amber") to refer to the property of attracting small objects after being rubbed.[3] This association gave rise to the English words "electric" and "electricity", which made their first appearance in print in Thomas Browne's *Pseudodoxia Epidemica* of 1646.[4]

Early investigators of the 18th century who suspected that the electrical force diminished with distance as the force of gravity did (i.e., as the inverse square of the distance) included Daniel Bernoulli[5] and Alessandro Volta, both of whom measured the force between plates of a capacitor, and Franz Aepinus who supposed the inverse-square law in 1758.[6]

Based on experiments with electrically charged spheres, Joseph Priestley of England was among the first to propose that electrical force followed an inverse-square law, similar to Newton's law of universal gravitation. However, he did not generalize or elaborate on this.[7] In 1767, he conjectured that the force between charges varied as the inverse square of the distance.[8][9]

In 1769, Scottish physicist John Robison announced that, according to his measurements, the force of repulsion between two spheres with charges of the same sign varied as $x^{-2.06}$.[10]

In the early 1770s, the dependence of the force between charged bodies upon both distance and charge had already been discovered, but not published, by Henry Cavendish of England.[11]

Finally, in 1785, the French physicist Charles-Augustin de Coulomb published his first three reports of electricity and magnetism where he stated his law. This publication was essential to the development of the theory of electromagnetism.[12] He used a torsion balance to study the repulsion and attraction forces of charged particles, and determined that the magnitude of the electric force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

The torsion balance consists of a bar suspended from its middle by a thin fiber. The fiber acts as a very weak torsion spring. In Coulomb's experiment, the torsion balance was an insulating rod with a metal-coated ball attached to one

Charles-Augustin de Coulomb

end, suspended by a silk thread. The ball was charged with a known charge of static electricity, and a second charged ball of the same polarity was brought near it. The two charged balls repelled one another, twisting the fiber through a certain angle, which could be read from a scale on the instrument. By knowing how much force it took to twist the fiber through a given angle, Coulomb was able to calculate the force between the balls and derive his inverse-square proportionality law.

3.2   The law

Coulomb's law states that:

> The magnitude of the electrostatic force of interaction between two point charges is directly propor-
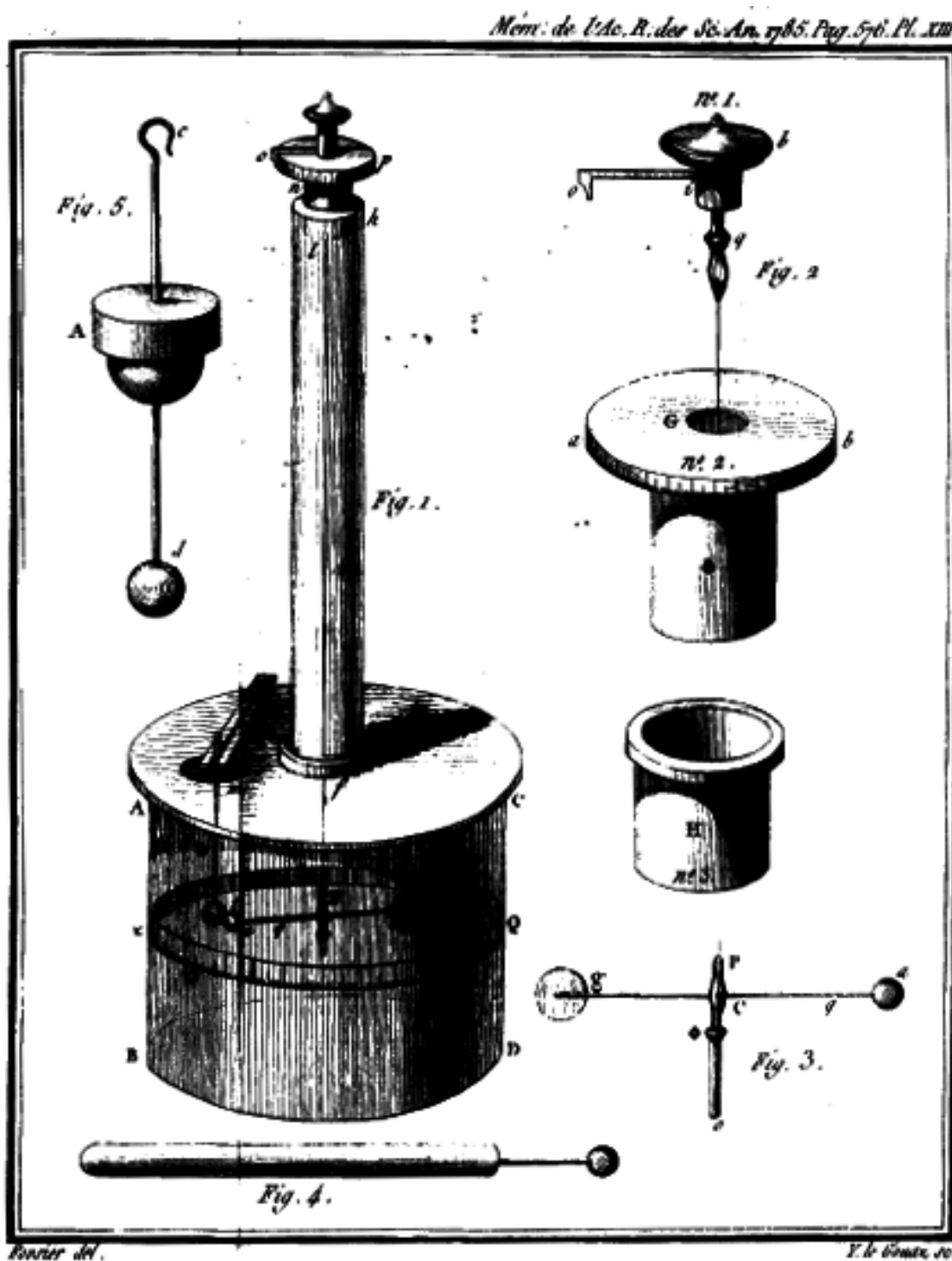
Coulomb's torsion balance

tional to the scalar multiplication of the magnitudes of charges and inversely proportional to the square of the distance between them.[12]

The force is along the straight line joining them. If the two charges have the same sign, the electro-static force between them is repulsive; if they have different signs, the force between them is attractive.

Coulomb's law can also be stated as a simple mathematical expression. The scalar and vector forms of the mathematical equation are

$$|\mathbf{F}| = k_e \frac{|q_1 q_2|}{r^2} \qquad \text{and} \qquad \mathbf{F}_1 = k_e \frac{q_1 q_2}{|\mathbf{r}_{21}|^2} \hat{\mathbf{r}}_{21}, \qquad \text{respectively,}$$
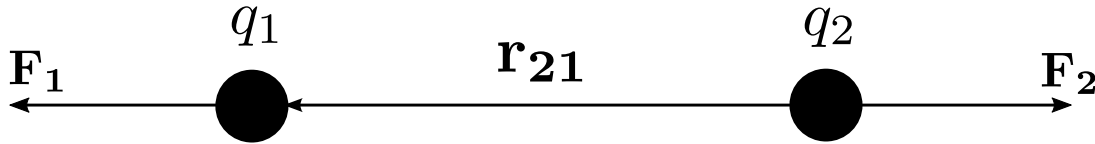
A graphical representation of Coulomb's law

where k_e is Coulomb's constant ($k_e = 8.987\,551\,787\,368\,176\,4 \times 10^9$ N · m² · C^{-2}), q_1 and q_2 are the signed magnitudes of the charges, the scalar r is the distance between the charges, the vector $\boldsymbol{r_{21}} = \boldsymbol{r_1} - \boldsymbol{r_2}$ is the vectorial distance between the charges, and $\hat{\boldsymbol{r}}_{21} = \boldsymbol{r_{21}}/|\boldsymbol{r_{21}}|$ (a unit vector pointing from q_2 to q_1). The vector form of the equation calculates the force $\mathbf{F}_1$ applied on q_1 by q_2 . If $\mathbf{r}_{12}$ is used instead, then the effect on q_2 can be found. It can be also calculated using Newton's third law: $\mathbf{F}_2 = -\mathbf{F}_1$.

3.2.1 Units

Electromagnetic theory is usually expressed using the standard SI units. Force is measured in newtons, charge in coulombs, and distance in metres. Coulomb's constant is given by $k_e = 1/(4\pi\varepsilon_0)$. The constant ε_0 is the permittivity of free space in C² m^{-2} N^{-1}. And ε is the relative permittivity of the material in which the charges are immersed, and is dimensionless.

The SI derived units for the electric field are volts per meter, newtons per coulomb, or tesla meters per second.

Coulomb's law and Coulomb's constant can also be interpreted in various terms:

- Atomic units. In atomic units the force is expressed in hartrees per Bohr radius, the charge in terms of the elementary charge, and the distances in terms of the *Bohr radius.*

- Electrostatic units or Gaussian units. In electrostatic units and Gaussian units, the unit charge (*esu* or statcoulomb) is defined in such a way that the Coulomb constant k disappears because it has the value of one and becomes dimensionless.

3.2.2 Electric field

An electric field is a vector field that associates to each point in space the Coulomb force experienced by a test charge. In the simplest case, the field is considered to be generated solely by a single source point charge. The strength and direction of the Coulomb force $\boldsymbol{F}$ on a test charge q_t depends on the electric field $\boldsymbol{E}$ that it finds itself in, such that $\boldsymbol{F} = q_t \boldsymbol{E}$. If the field is generated by a positive source point charge q , the direction of the electric field points along lines directed radially outwards from it, i.e. in the direction that a positive point test charge q_t would move if placed in the field. For a negative point source charge, the direction is radially inwards.

The magnitude of the electric field $\boldsymbol{E}$ can be derived from Coulomb's law. By choosing one of the point charges to be the source, and the other to be the test charge, it follows from Coulomb's law that the magnitude of the electric field $\boldsymbol{E}$ created by a single source point charge q at a certain distance from it r in vacuum is given by:
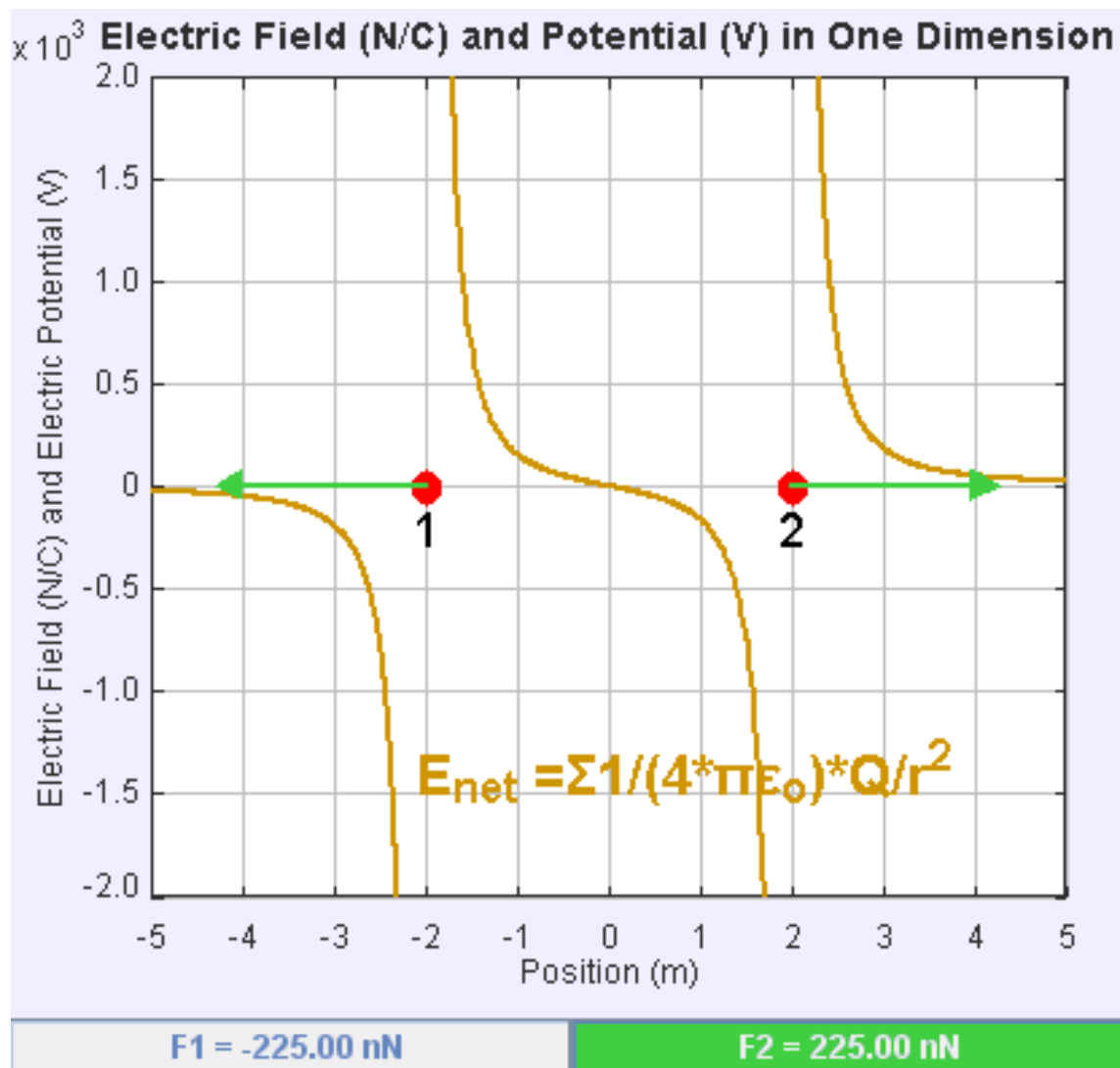
$$|\boldsymbol{E}| = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}$$

3.2.3 Coulomb's constant

Main article: Coulomb's constant

Coulomb's constant is a proportionality factor that appears in Coulomb's law as well as in other electric-related formulas. Denoted k_e , it is also called the electric force constant or electrostatic constant, hence the subscript e .

The exact value of Coulomb's constant is:

If the two charges have the same sign, the electrostatic force between them is repulsive; if they have different sign, the force between them is attractive.
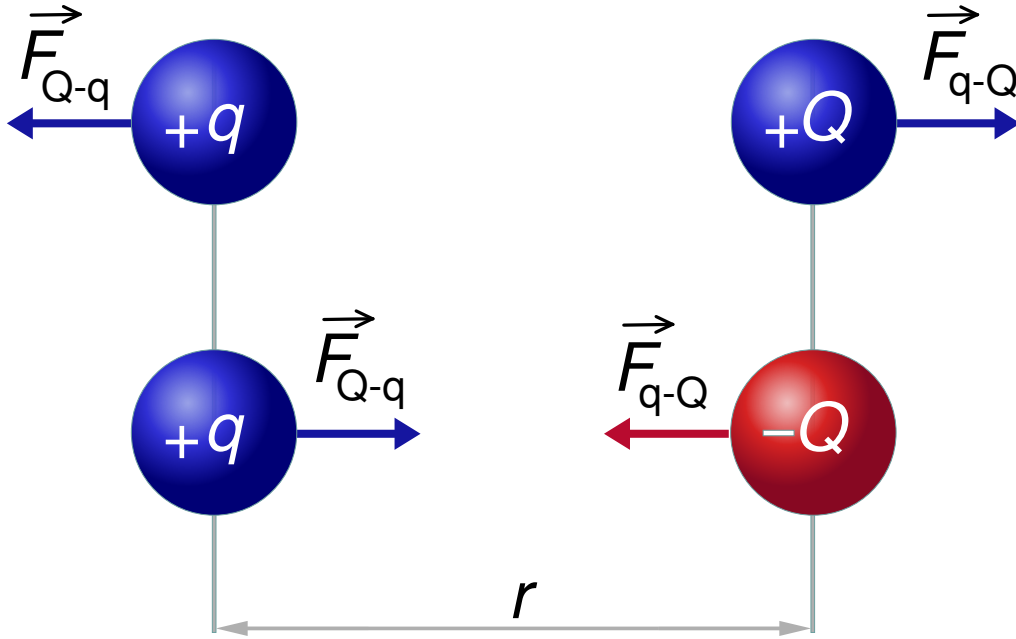
$$k_e = \frac{1}{4\pi\varepsilon_0} = \frac{c_0^2\mu_0}{4\pi} = c_0^2 \times 10^{-7} \text{ H} \cdot \text{m}^{-1}$$
$$= 8.987\,551\,787\,368\,176\,4 \times 10^9 \text{ N} \cdot \text{m}^2 \cdot \text{C}^{-2}$$

3.2.4 Conditions for validity

There are three conditions to be fulfilled for the validity of Coulomb's law:

1. The charges considered must be point charges.

2. They should be stationary with respect to each other.

3. The two point charges should be placed in a single medium.

$$\left| \overrightarrow{F_{Q\text{-}q}} \right| = \left| \overrightarrow{F_{q\text{-}Q}} \right| = k \, \frac{|q \times Q|}{r^2}$$

The absolute value of the force **F** *between two point charges* q *and* Q *relates to the distance between the point charges and to the simple product of their charges. The diagram shows that like charges repel each other, and opposite charges attract each other.*

3.3   Scalar form

When it is only of interest to know the magnitude of the electrostatic force (and not its direction), it may be easiest to consider a scalar version of the law. The scalar form of Coulomb's Law relates the magnitude and sign of the electrostatic force **F** acting simultaneously on two point charges q_1 and q_2 as follows:

$$|\boldsymbol{F}| = k_e \frac{|q_1 q_2|}{r^2}$$

where r is the separation distance and k_e is Coulomb's constant. If the product $q_1 q_2$ is positive, the force between the two charges is repulsive; if the product is negative, the force between them is attractive.[13]

3.4   Vector form

Coulomb's law states that the electrostatic force $\boldsymbol{F_1}$ experienced by a charge, q_1 at position $\boldsymbol{r_1}$, in the vicinity of another charge, q_2 at position $\boldsymbol{r_2}$, in a vacuum is equal to:

$$\boldsymbol{F_1} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{(\boldsymbol{r_1} - \boldsymbol{r_2})}{|\boldsymbol{r_1} - \boldsymbol{r_2}|^3} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\hat{\boldsymbol{r}}_{21}}{|\boldsymbol{r_{21}}|^2},$$

where $\boldsymbol{r_{21}} = \boldsymbol{r_1} - \boldsymbol{r_2}$, the unit vector $\hat{\boldsymbol{r}}_{21} = \boldsymbol{r_{21}}/|\boldsymbol{r_{21}}|$, and ε_0 is the electric constant.

The vector form of Coulomb's law is simply the scalar definition of the law with the direction given by the unit vector, $\hat{\boldsymbol{r}}_{21}$, parallel with the line *from* charge q_2 *to* charge q_1 .[14] If both charges have the same sign (like charges) then
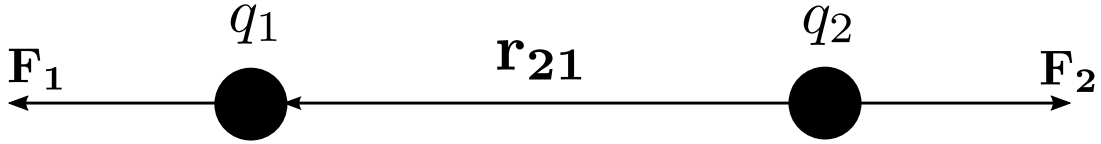
In the image, the vector $\boldsymbol{F}_1$ is the force experienced by q_1 , and the vector $\boldsymbol{F}_2$ is the force experienced by q_2 . When $q_1 q_2 > 0$ the forces are repulsive (as in the image) and when $q_1 q_2 < 0$ the forces are attractive (opposite to the image). The magnitude of the forces will always be equal.

the product $q_1 q_2$ is positive and the direction of the force on q_1 is given by $\hat{\boldsymbol{r}}_{21}$; the charges repel each other. If the charges have opposite signs then the product $q_1 q_2$ is negative and the direction of the force on q_1 is given by $-\hat{\boldsymbol{r}}_{21}$; the charges attract each other.

The electrostatic force $\boldsymbol{F_2}$ experienced by q_2 , according to Newton's third law, is $\boldsymbol{F_2} = -\boldsymbol{F_1}$.

3.4.1 System of discrete charges

The law of superposition allows Coulomb's law to be extended to include any number of point charges. The force acting on a point charge due to a system of point charges is simply the vector addition of the individual forces acting alone on that point charge due to each one of the charges. The resulting force vector is parallel to the electric field vector at that point, with that point charge removed.

The force $\boldsymbol{F}$ on a small charge, q at position $\boldsymbol{r}$, due to a system of N discrete charges in vacuum is:

$$\boldsymbol{F}(\boldsymbol{r}) = \frac{q}{4\pi\varepsilon_0} \sum_{i=1}^{N} q_i \frac{\boldsymbol{r} - \boldsymbol{r_i}}{|\boldsymbol{r} - \boldsymbol{r_i}|^3} = \frac{q}{4\pi\varepsilon_0} \sum_{i=1}^{N} q_i \frac{\widehat{\boldsymbol{R_i}}}{|\boldsymbol{R_i}|^2},$$

where q_i and $\boldsymbol{r_i}$ are the magnitude and position respectively of the ith charge, $\widehat{\boldsymbol{R_i}}$ is a unit vector in the direction of $\boldsymbol{R_i} = \boldsymbol{r} - \boldsymbol{r_i}$ (a vector pointing from charges q_i to q).[14]

3.4.2 Continuous charge distribution

In this case, the principle of linear superposition is also used. For a continuous charge distribution, an integral over the region containing the charge is equivalent to an infinite summation, treating each infinitesimal element of space as a point charge dq . The distribution of charge is usually linear, surface or volumetric.

For a linear charge distribution (a good approximation for charge in a wire) where $\lambda(\boldsymbol{r'})$ gives the charge per unit length at position $\boldsymbol{r'}$, and dl' is an infinitesimal element of length,

$$dq = \lambda(\boldsymbol{r'})dl' \ .^{[15]}$$

For a surface charge distribution (a good approximation for charge on a plate in a parallel plate capacitor) where $\sigma(\boldsymbol{r'})$ gives the charge per unit area at position $\boldsymbol{r'}$, and dA' is an infinitesimal element of area,

$$dq = \sigma(\boldsymbol{r'}) \, dA'.$$

For a volume charge distribution (such as charge within a bulk metal) where $\rho(\boldsymbol{r'})$ gives the charge per unit volume at position $\boldsymbol{r'}$, and dV' is an infinitesimal element of volume,

$$dq = \rho(\boldsymbol{r'}) \, dV'. \ ^{[14]}$$

The force on a small test charge q' at position $\boldsymbol{r}$ in vacuum is given by the integral over the distribution of charge:

$$\boldsymbol{F} = \frac{q'}{4\pi\varepsilon_0} \int dq \frac{\boldsymbol{r} - \boldsymbol{r'}}{|\boldsymbol{r} - \boldsymbol{r'}|^3}.$$

3.5   Simple experiment to verify Coulomb's law
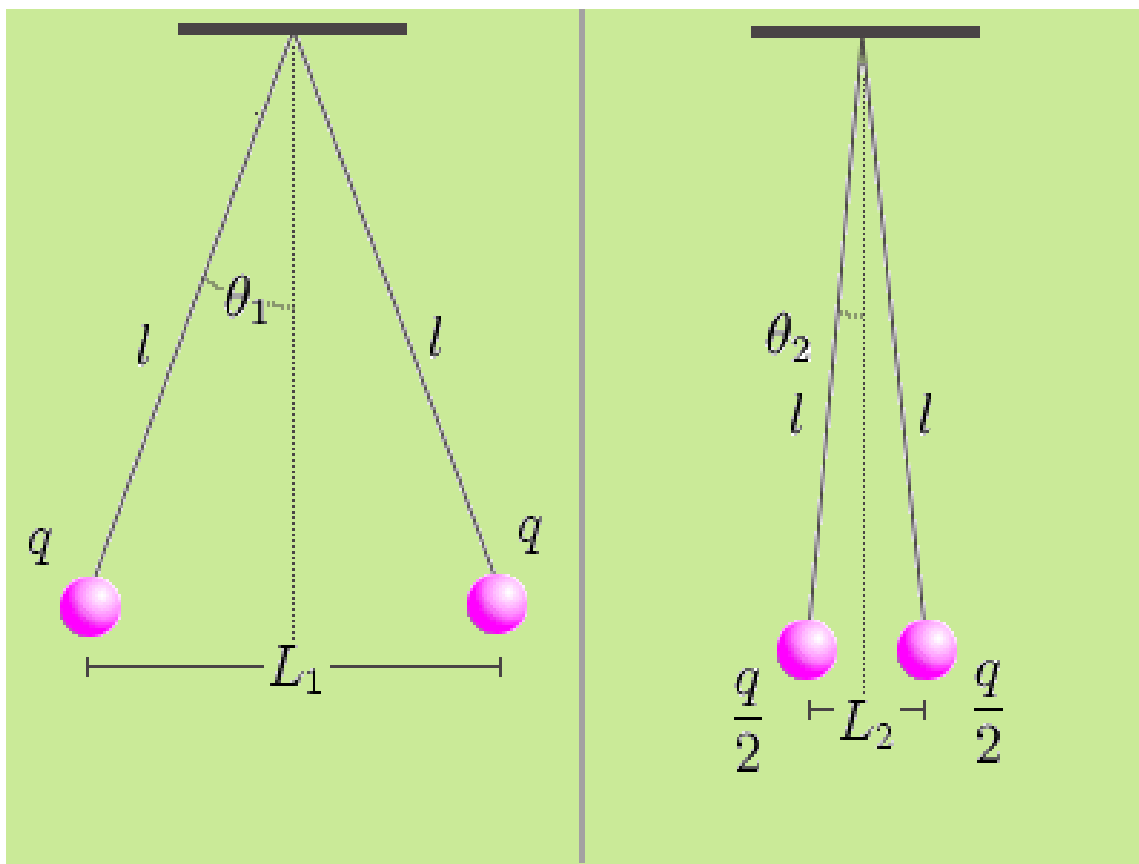
Experiment to verify Coulomb's law.

It is possible to verify Coulomb's law with a simple experiment. Let's consider two small spheres of mass m and same-sign charge q , hanging from two ropes of negligible mass of length l . The forces acting on each sphere are three: the weight mg , the rope tension T and the electric force $\boldsymbol{F}$.

In the equilibrium state:

and:

Dividing (**1**) by (**2**):

Being L_1 the distance between the charged spheres; the repulsion force between them F_1 , assuming Coulomb's law is correct, is equal to

so:

If we now discharge one of the spheres, and we put it in contact with the charged sphere, each one of them acquires a charge $q/2$. In the equilibrium state, the distance between the charges will be $L_2 < L_1$ and the repulsion force between them will be:

We know that $F_2 = mg.\tan\theta_2$. And:

$$\frac{\frac{q^2}{4}}{4\pi\epsilon_0 L_2^2} = mg.\tan\theta_2$$

Dividing (**4**) by (**5**), we get:

Measuring the angles θ_1 and θ_2 and the distance between the charges L_1 and L_2 is sufficient to verify that the equality is true taking into account the experimental error. In practice, angles can be difficult to measure, so if the length of the ropes is sufficiently great, the angles will be small enough to make the following approximation:

Using this approximation, the relationship (**6**) becomes the much simpler expression:

In this way, the verification is limited to measuring the distance between the charges and check that the division approximates the theoretical value.

3.6 Electrostatic approximation

In either formulation, Coulomb's law is fully accurate only when the objects are stationary, and remains approximately correct only for slow movement. These conditions are collectively known as the electrostatic approximation. When movement takes place, magnetic fields that alter the force on the two objects are produced. The magnetic interaction between moving charges may be thought of as a manifestation of the force from the electrostatic field but with Einstein's theory of relativity taken into consideration.

3.6.1 Atomic forces

Coulomb's law holds even within atoms, correctly describing the force between the positively charged atomic nucleus and each of the negatively charged electrons. This simple law also correctly accounts for the forces that bind atoms together to form molecules and for the forces that bind atoms and molecules together to form solids and liquids. Generally, as the distance between ions increases, the energy of attraction approaches zero and ionic bonding is less favorable. As the magnitude of opposing charges increases, energy increases and ionic bonding is more favorable.

3.7 See also

- Biot–Savart law

- Gauss's law

- Method of image charges

- Electromagnetic force

- Molecular modelling

- Static forces and virtual-particle exchange

- Darwin Lagrangian

- Newton's law of universal gravitation, which uses a similar structure, but for mass instead of charge.

3.8 Notes

[1] Stewart, Joseph (2001). *Intermediate Electromagnetic Theory*. World Scientific. p. 50. ISBN 981-02-4471-1

[2] Simpson, Brian (2003). *Electrical Stimulation and the Relief of Pain*. Elsevier Health Sciences. pp. 6–7. ISBN 0-444-51258-6

[3] Baigrie, Brian (2006). *Electricity and Magnetism: A Historical Perspective*. Greenwood Press. pp. 7–8. ISBN 0-313-33358-0

[4] Chalmers, Gordon (1937). "The Lodestone and the Understanding of Matter in Seventeenth Century England". *Philosophy of Science* **4** (1): 75–95. doi:10.1086/286445

[5] Socin, Abel (1760). *Acta Helvetica Physico-Mathematico-Anatomico-Botanico-Medica* (in Latin) **4**. Basileae. pp. 224, 225.

[6] Heilbron, J.L. (1979). *Electricity in the 17th and 18th Centuries: A Study of Early Modern Physics*. Los Angeles, California: University of California Press. pp. 460–462 and 464 (including footnote 44). ISBN 0486406881.

[7] Schofield, Robert E. (1997). *The Enlightenment of Joseph Priestley: A Study of his Life and Work from 1733 to 1773*. University Park: Pennsylvania State University Press. pp. 144–56. ISBN 0-271-01662-0.

[8] Priestley, Joseph (1767). *The History and Present State of Electricity, with Original Experiments*. London, England. p. 732.

> May we not infer from this experiment, that the attraction of electricity is subject to the same laws with that of gravitation, and is therefore according to the squares of the distances; since it is easily demonstrated, that were the earth in the form of a shell, a body in the inside of it would not be attracted to one side more than another?

[9] Elliott, Robert S. (1999). *Electromagnetics: History, Theory, and Applications*. ISBN 978-0-7803-5384-8.

[10] Robison, John (1822). Murray, John, ed. *A System of Mechanical Philosophy* **4**. London, England.
On page 68, the author states that in 1769 he announced his findings regarding the force between spheres of like charge.
On page 73, the author states the force between spheres of like charge varies as $x^{-2.06}$:

> The result of the whole was, that the mutual repulsion of two spheres, electrified positively or negatively, was very nearly in the inverse proportion of the squares of the distances of their centres, or rather in a proportion somewhat greater, approaching to $x^{-2.06}$.

When making experiments with charged spheres of opposite charge the results were similar, as stated on page 73:

> When the experiments were repeated with balls having opposite electricities, and which therefore attracted each other, the results were not altogether so regular and a few irregularities amounted to 1/6 of the whole; but these anomalies were as often on one side of the medium as on the other. This series of experiments gave a result which deviated as little as the former (or rather less) from the inverse duplicate ratio of the distances; but the deviation was in defect as the other was in excess.

Nonetheless, on page 74 the author infers that the actual action is related exactly to the inverse duplicate of the distance:

> We therefore think that it may be concluded, that the action between two spheres is exactly in the inverse duplicate ratio of the distance of their centres, and that this difference between the observed attractions and repulsions is owing to some unperceived cause in the form of the experiment.

On page 75, the authour compares the electric and gravitational forces:

> Therefore we may conclude, that the law of electric attraction and repulsion is similar to that of gravitation, and that each of those forces diminishes in the same proportion that the square of the distance between the particles increases.

[11] Maxwell, James Clerk, ed. (1967) [1879]. "Experiments on Electricity: Experimental determination of the law of electric force.". *The Electrical Researches of the Honourable Henry Cavendish...* (1st ed.). Cambridge, England: Cambridge University Press. pp. 104–113.
On pages 111 and 112 the author states:

> We may therefore conclude that the electric attraction and repulsion must be inversely as some power of the distance between that of the 2 + 1/50 th and that of the 2 - 1/50 th, and there is no reason to think that it differs at all from the inverse duplicate ratio.

[12] Coulomb (1785a) "Premier mémoire sur l'électricité et le magnétisme," *Histoire de l'Académie Royale des Sciences*, pages 569-577 — Coulomb studied the repulsive force between bodies having electrical charges of the same sign:

> *Il résulte donc de ces trois essais, que l'action répulsive que les deux balles électrifées de la même nature d'électricité exercent l'une sur l'autre, suit la raison inverse du carré des distances.* Translation: It follows therefore from these three tests, that the repulsive force that the two balls — [that were] electrified with the same kind of electricity — exert on each other, follows the inverse proportion of the square of the distance.

Coulomb also showed that oppositely charged bodies obey an inverse-square law of attraction.

[13] Coulomb's law, Hyperphysics

[14] Coulomb's law, University of Texas

[15] Charged rods, PhysicsLab.org

3.9 References

- Coulomb, Charles Augustin (1788) [1785]. "Premier mémoire sur l'électricité et le magnétisme". *Histoire de l'Académie Royale des Sciences*. Imprimerie Royale. pp. 569–577.

- Coulomb, Charles Augustin (1788) [1785]. "Second mémoire sur l'électricité et le magnétisme". *Histoire de l'Académie Royale des Sciences*. Imprimerie Royale. pp. 578–611.

- Griffiths, David J. (1998). *Introduction to Electrodynamics* (3rd ed.). Prentice Hall. ISBN 0-13-805326-X.

- Tipler, Paul A.; Mosca, Gene (2008). *Physics for Scientists and Engineers* (6th ed.). New York: W. H. Freeman and Company. ISBN 0-7167-8964-7. LCCN 2007010418.

- Young, Hugh D.; Freedman, Roger A. (2010). *Sears and Zemansky's University Physics : With Modern Physics* (13th ed.). Addison-Wesley (Pearson). ISBN 978-0-321-69686-1.

3.10 External links

- *Coulomb's Law* on Project PHYSNET

- Electricity and the Atom—a chapter from an online textbook

- A maze game for teaching Coulomb's Law—a game created by the Molecular Workbench software

- Electric Charges, Polarization, Electric Force, Coulomb's Law Walter Lewin, *8.02 Electricity and Magnetism, Spring 2002: Lecture 1* (video). MIT OpenCourseWare. License: Creative Commons Attribution-Noncommercial-Share Alike.

Chapter 4

Electric field
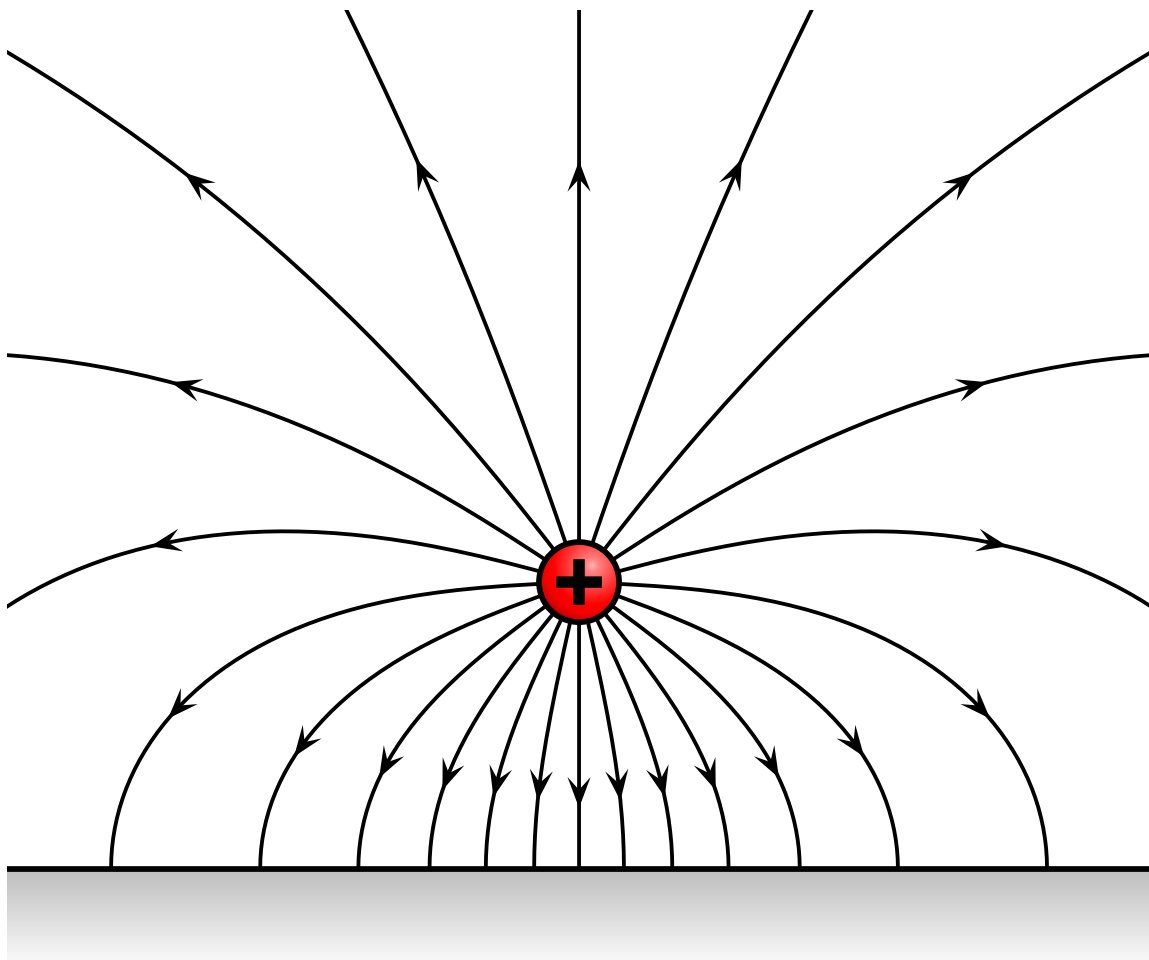
Electric field lines emanating from a point positive electric charge suspended over an infinite sheet of conducting material.

The **electric field** is a component of the electromagnetic field. It is a vector field, and it is generated by electric charges or time-varying magnetic fields as described by Maxwell's equations.[1] The concept of an electric field was introduced by Michael Faraday.[2]

4.1 Definition

The electric field **E** at a given point is defined as the (vectorial) force **F** that would be exerted on a stationary test particle of unit charge by electromagnetic forces (i.e. the Lorentz force). A particle of charge q would be subject to

a force $\mathbf{F} = q \cdot \mathbf{E}$.

Its SI units are newtons per coulomb (N·C^{-1}) or, equivalently, volts per metre (V·m^{-1}), which in terms of SI base units are kg·m·s^{-3}·A^{-1} .

4.2 Sources of electric field

4.2.1 Causes and description

Electric fields are caused by electric charges or varying magnetic fields. The former effect is described by Gauss's law, the latter by Faraday's law of induction, which together are enough to define the behavior of the electric field as a function of charge repartition and magnetic field. However, since the magnetic field is described as a function of electric field, the equations of both fields are coupled and together form Maxwell's equations that describe both fields as a function of charges and currents.

In the special case of a steady state (stationary charges and currents), the Maxwell-Faraday inductive effect disappears. The resulting two equations (Gauss's law $\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$ and Faraday's law with no induction term $\nabla \times \mathbf{E} = 0$), taken together, are equivalent to Coulomb's law, written as $\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0} \int d\boldsymbol{r}' \rho(\boldsymbol{r}') \frac{\boldsymbol{r}-\boldsymbol{r}'}{|\boldsymbol{r}-\boldsymbol{r}'|^3}$ for a charge density $\rho(\mathbf{r})$ ($\mathbf{r}$ denotes the position in space). Notice that ε_0 , the permittivity of vacuum, must be substituted if charges are considered in non-empty media.

4.2.2 Continuous vs. discrete charge repartition

Main article: Charge density

The equations of electromagnetism are best described in a continuous description. However, charges are sometimes best described as discrete points; for example, some models may describe electrons as punctual sources where charge density is infinite on an infinitesimal section of space.

A charge q located in $\mathbf{r_0}$ can be described mathematically as a charge density $\rho(\mathbf{r}) = q\delta(\mathbf{r} - \mathbf{r_0})$, where the Dirac delta function (in three dimensions) is used. Conversely, a charge distribution can be approximated by many small punctual charges.

4.3 Superposition principle

Electric fields satisfy the superposition principle, because Maxwell's equations are linear. As a result, if $\mathbf{E}_1$ and $\mathbf{E}_2$ are the electric fields resulting from distribution of charges ρ_1 and ρ_2 , a distribution of charges $\rho_1 + \rho_2$ will create an electric field $\mathbf{E}_1 + \mathbf{E}_2$; for instance, Coulomb's law is linear in charge density as well.

This principle is useful to calculate the field created by multiple point charges. If charges $q_1, q_2, ..., q_n$ are stationary in space at $\mathbf{r}_1, \mathbf{r}_2, ...\mathbf{r}_n$, in the absence of currents, the superposition principle proves that the resulting field is the sum of fields generated by each particle as described by Coulomb's law:

$$\mathbf{E}(\mathbf{r}) = \sum_{i=1}^{N} \mathbf{E}_i(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} q_i \frac{\mathbf{r} - \mathbf{r}_i}{|\mathbf{r} - \mathbf{r}_i|^3}$$

4.4 Electrostatic fields

Main article: Electrostatics

Electrostatic fields are $\mathbf{E}$-fields which do not change with time, which happens when charges and currents are stationary. In that case, Coulomb's law fully describes the field.
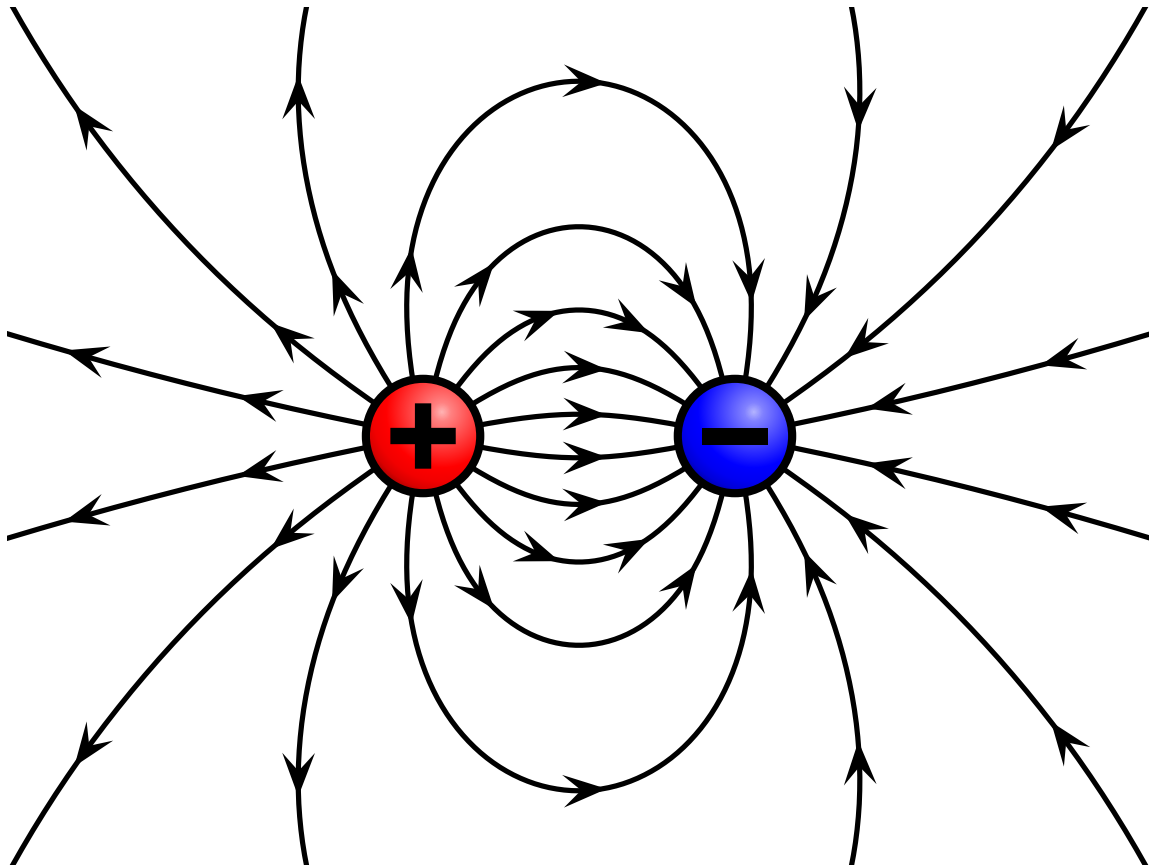
Illustration of the electric field surrounding a positive (red) and a negative (blue) charge.

4.4.1 Electric potential

Main article: Conservative vector field § Irrotational vector fields

By Faraday's law, the electric field has zero curl. One can then define an electric potential, that is, a function Φ such that $\mathbf{E} = -\nabla\Phi$.[3] This is analogous to the gravitational potential.

4.4.2 Parallels between electrostatic and gravitational fields

Coulomb's law, which describes the interaction of electric charges:

$$\mathbf{F} = q\left(\frac{Q}{4\pi\varepsilon_0}\frac{\hat{\mathbf{r}}}{|\mathbf{r}|^2}\right) = q\mathbf{E}$$

is similar to Newton's law of universal gravitation:

$$\mathbf{F} = m\left(-GM\frac{\hat{\mathbf{r}}}{|\mathbf{r}|^2}\right) = m\mathbf{g}$$

(where $\hat{\mathbf{r}} = \frac{\mathbf{r}}{|\mathbf{r}|}$).

This suggests similarities between the electric field $\mathbf{E}$ and the gravitational field $\mathbf{g}$, or their associated potentials. Mass is sometimes called "gravitational charge" because of that similarity.

Electrostatic and gravitational forces both are central, conservative and obey an inverse-square law.
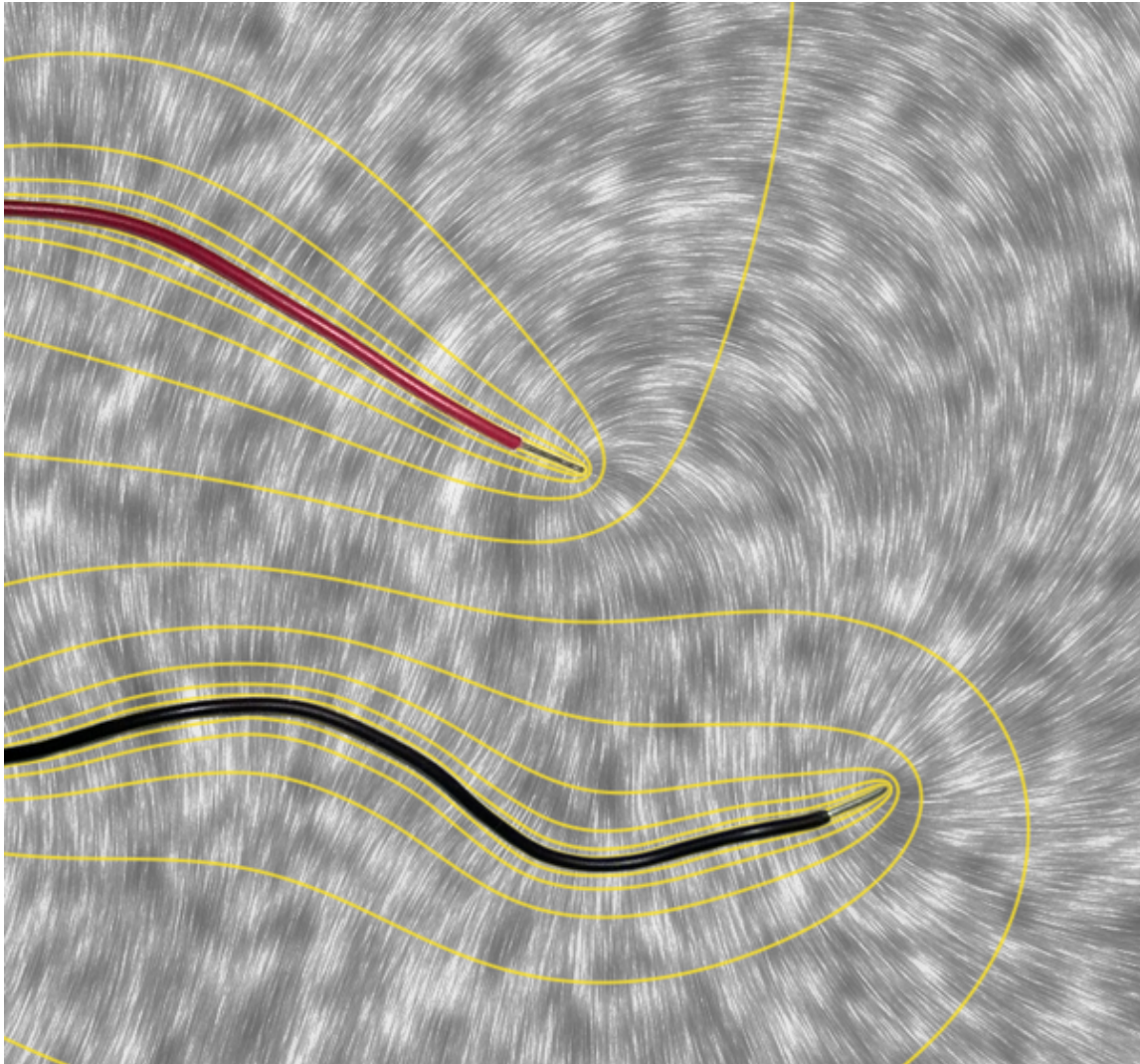
Electric field between two conductors

4.4.3   Uniform fields

A uniform field is one in which the electric field is constant at every point. It can be approximated by placing two conducting plates parallel to each other and maintaining a voltage (potential difference) between them; it is only an approximation because of boundary effects (near the edge of the planes, electric field is distorted because the plane does not continue). Assuming infinite planes, the magnitude of the electric field E is:

$$E = -\frac{\Delta\phi}{d}$$

where $\Delta\phi$ is the potential difference between the plates and d is the distance separating the plates. The negative sign arises as positive charges repel, so a positive charge will experience a force away from the positively charged plate, in the opposite direction to that in which the voltage increases. In micro- and nanoapplications, for instance in relation to semiconductors, a typical magnitude of an electric field is in the order of 10^6 V·m^{-1}, achieved by applying a voltage of the order of 1 volt between conductors spaced 1 μm apart.

4.5   Electrodynamic fields

Main article: Electrodynamics

Electrodynamic fields are **E**-fields which do change with time, for instance when charges are in motion.

The electric field cannot be described independently of the magnetic field in that case. If **A** is the magnetic vector potential, defined so that $\mathbf{B} = \nabla \times \mathbf{A}$, one can still define an electric potential Φ such that:

$$\mathbf{E} = -\nabla\Phi - \frac{\partial \mathbf{A}}{\partial t}$$

One can recover Faraday's law of induction by taking the curl of that equation

[4]

$$\nabla \times \mathbf{E} = -\frac{\partial(\nabla \times \mathbf{A})}{\partial t} = -\frac{\partial \mathbf{B}}{\partial t}$$

which justifies, a posteriori, the previous form for **E**.

4.6   Energy in the electric field

If the magnetic field **B** is nonzero,

The total energy per unit volume stored by the electromagnetic field is[5]

$$u_{EM} = \frac{\varepsilon}{2}|\mathbf{E}|^2 + \frac{1}{2\mu}|\mathbf{B}|^2$$

where ε is the permittivity of the medium in which the field exists, μ its magnetic permeability, and **E** and **B** are the electric and magnetic field vectors.

As **E** and **B** fields are coupled, it would be misleading to split this expression into "electric" and "magnetic" contributions. However, in the steady-state case, the fields are no longer coupled (see Maxwell's equations). It makes sense in that case to compute the electrostatic energy per unit volume:

$$u_{ES} = \frac{1}{2}\varepsilon|\mathbf{E}|^2 \, ,$$

The total energy U stored in the electric field in a given volume V is therefore

$$U_{ES} = \frac{1}{2}\varepsilon \int_V |\mathbf{E}|^2 \, \mathrm{d}V \, ,$$

4.7   Further extensions

4.7.1   Definitive equation of vector fields

See also: Defining equation (physics) and List of electromagnetism equations

In the presence of matter, it is helpful in electromagnetism to extend the notion of the electric field into three vector fields, rather than just one:[6]

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$$

where **P** is the electric polarization – the volume density of electric dipole moments, and **D** is the electric displacement field. Since **E** and **P** are defined separately, this equation can be used to define **D**. The physical interpretation of **D** is not as clear as **E** (effectively the field applied to the material) or **P** (induced field due to the dipoles in the material), but still serves as a convenient mathematical simplification, since Maxwell's equations can be simplified in terms of free charges and currents.

4.7.2 Constitutive relation

Main article: Constitutive equation

The **E** and **D** fields are related by the permittivity of the material, ε.[7][8]

For linear, homogeneous, isotropic materials **E** and **D** are proportional and constant throughout the region, there is no position dependence: For inhomogeneous materials, there is a position dependence throughout the material:

$$\mathbf{D}(\mathbf{r}) = \varepsilon \mathbf{E}(\mathbf{r})$$

For anisotropic materials the **E** and **D** fields are not parallel, and so **E** and **D** are related by the permittivity tensor (a 2nd order tensor field), in component form:

$$D_i = \varepsilon_{ij} E_j$$

For non-linear media, **E** and **D** are not proportional. Materials can have varying extents of linearity, homogeneity and isotropy.

4.8 See also

- Classical electromagnetism

- Field strength

- signal strength in telecommunications

- Magnetism

- Teltron Tube

- Teledeltos, a conductive paper that may be used as a simple analog computer for modelling fields.

4.9 References

[1] Richard Feynman (1970). *The Feynman Lectures on Physics Vol II*. Addison Wesley Longman. ISBN 978-0-201-02115-8.

[2] http://public.wsu.edu/~{}jtd/Physics206/michael_faraday.htm

[3] http://physicspages.com/2011/10/08/curl-potential-in-electrostatics/

[4] Huray, Paul G. (2009). *Maxwell's Equations*. Wiley-IEEE. p. 205. ISBN 0-470-54276-4.

[5] Introduction to Electrodynamics (3rd Edition), D.J. Griffiths, Pearson Education, Dorling Kindersley, 2007, ISBN 81-7758-293-3

[6] Electromagnetism (2nd Edition), I.S. Grant, W.R. Phillips, Manchester Physics, John Wiley & Sons, 2008, ISBN 978-0-471-92712-9

[7] Electricity and Modern Physics (2nd Edition), G.A.G. Bennet, Edward Arnold (UK), 1974, ISBN 0-7131-2459-8

[8] Electromagnetism (2nd Edition), I.S. Grant, W.R. Phillips, Manchester Physics, John Wiley & Sons, 2008, ISBN 978-0-471-92712-9

4.10   External links

- Electric field in "Electricity and Magnetism", R Nave – Hyperphysics, Georgia State University

- 'Gauss's Law' – Chapter 24 of Frank Wolfs's lectures at University of Rochester

- 'The Electric Field' – Chapter 23 of Frank Wolfs's lectures at University of Rochester

- – An applet that shows the electric field of a moving point charge.

- Fields – a chapter from an online textbook

- Learning by Simulations Interactive simulation of an electric field of up to four point charges

- Java simulations of electrostatics in 2-D and 3-D

- Interactive Flash simulation picturing the electric field of user-defined or preselected sets of point charges by field vectors, field lines, or equipotential lines. Author: David Chappell

Chapter 5

Electric flux

In electromagnetism, **electric flux** is the measure of flow of the electric field through a given area. Electric flux is proportional to the number of electric field lines going through a normally perpendicular surface. If the electric field is uniform, the electric flux passing through a surface of vector area **S** is

$$\Phi_E = \mathbf{E} \cdot \mathbf{S} = ES\cos\theta,$$

where **E** is the electric field (having units of V/m), E is its magnitude, S is the area of the surface, and θ is the angle between the electric field lines and the normal (perpendicular) to S.

For a non-uniform electric field, the electric flux $d\Phi E$ through a small surface area $d\mathbf{S}$ is given by

$$d\Phi_E = \mathbf{E} \cdot d\mathbf{S}$$

(the electric field, **E**, multiplied by the component of area perpendicular to the field). The electric flux over a surface S is therefore given by the surface integral:

$$\Phi_E = \iint_S \mathbf{E} \cdot d\mathbf{S}$$

where **E** is the electric field and $d\mathbf{S}$ is a differential area on the closed surface S with an outward facing surface normal defining its direction.

For a closed Gaussian surface, electric flux is given by:

$$\Phi_E = \oiint_S \mathbf{E} \cdot d\mathbf{S} = \frac{Q}{\epsilon_0}$$

where

> **E** is the electric field,
>
> S is any closed surface,
>
> Q is the total electric charge inside the surface S,
>
> ε_0 is the electric constant (a universal constant, also called the "permittivity of free space") ($\varepsilon_0 \approx 8.854\ 187\ 817...$ x 10^{-12} farads per meter (F·m^{-1})).

This relation is known as Gauss' law for electric field in its integral form and it is one of the four Maxwell's equations.

While the electric flux is not affected by charges that are not within the closed surface, the net electric field, **E**, in the Gauss' Law equation, can be affected by charges that lie outside the closed surface. While Gauss' Law holds for

all situations, it is only useful for "by hand" calculations when high degrees of symmetry exist in the electric field. Examples include spherical and cylindrical symmetry.

Electrical flux has SI units of volt metres ($V\ m$), or, equivalently, newton metres squared per coulomb ($N\ m^2\ C^{-1}$). Thus, the SI base units of electric flux are $kg{\cdot}m^3{\cdot}s^{-3}{\cdot}A^{-1}$.

Its dimensional formula is [$L^3MT^{-3}I^{-1}$].

5.1 See also

- Magnetic flux

- Maxwell's equations

related websites are following: http://www.citycollegiate.com/coulomb4_XII.htm[1]

5.2 References

[1] electric flux

5.3 External links

- Electric flux — HyperPhysics

Chapter 6

Field line

This article is about the modern use of "field lines" as a way to depict electromagnetic and other vector fields. For the role of these lines in the early history and philosophy of electromagnetism, see Line of force.

A **field line** is a locus that is defined by a vector field and a starting location within the field. Field lines are useful for
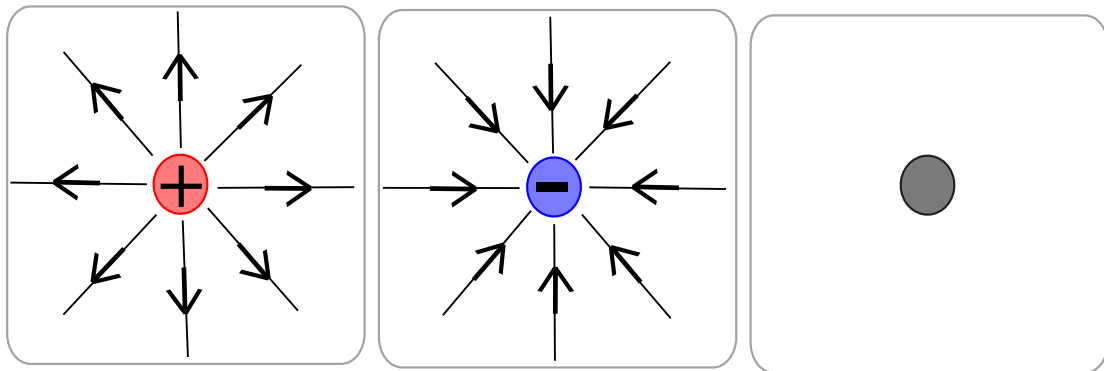
Field lines depicting the electric field created by a positive charge (left), negative charge (center), and uncharged object (right).
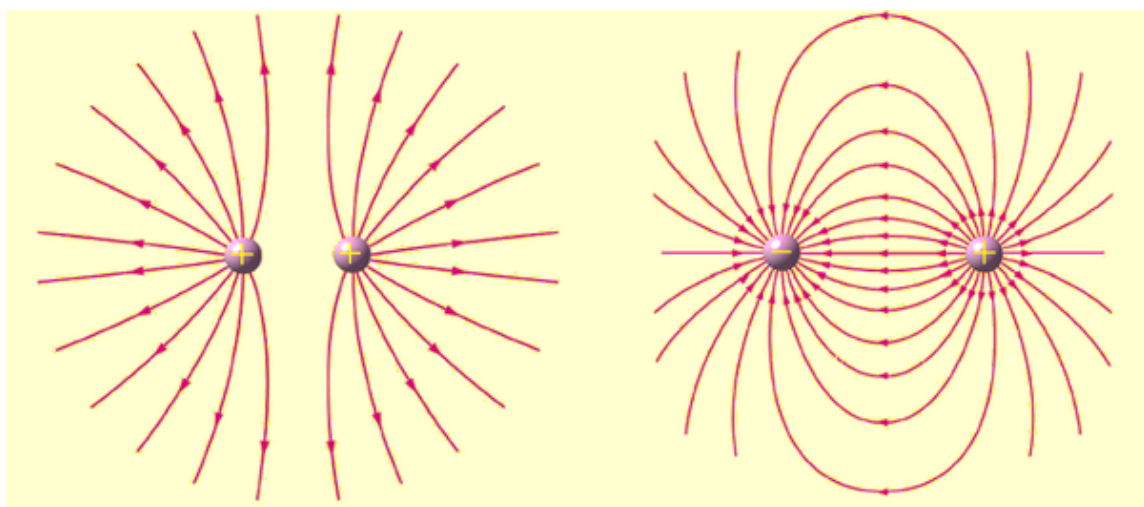
The figure at left shows the electric field lines of two equal positive charges. The figure at right shows the electric field lines of a dipole.

visualizing vector fields, which are otherwise hard to depict. Note that, like longitude and latitude lines on a globe, or

topographic lines on a topographic map, these lines are not physical lines that are actually present at certain locations; they are merely visualization tools.

6.1   Precise definition

A vector field defines a direction at all points in space; a field line for that vector field may be constructed by tracing a topographic path in the direction of the vector field. More precisely, the tangent line to the path at each point is required to be parallel to the vector field at that point.

A complete description of the geometry of all the field lines of a vector field is sufficient to completely specify the *direction* of the vector field everywhere. In order to also depict the *magnitude*, a selection of field lines is drawn such that the density of field lines (number of field lines per unit perpendicular area) at any location is proportional to the magnitude of the vector field at that point.

As a result of the divergence theorem, field lines start at *sources* and end at *sinks* of the vector field. (A "source" is wherever the divergence of the vector field is positive, a "sink" is wherever it is negative.) In physics, drawings of field lines are mainly useful in cases where the sources and sinks, if any, have a physical meaning, as opposed to e.g. the case of a force field of a radial harmonic.

For example, Gauss's law states that an electric field has sources at positive charges, sinks at negative charges, and neither elsewhere, so electric field lines start at positive charges and end at negative charges. (They can also potentially form closed loops, or extend to or from infinity, or continuing forever without closing in on itself). A gravitational field has no sources, it has sinks at masses, and it has neither elsewhere, gravitational field lines come from infinity and end at masses. A magnetic field has no sources or sinks (Gauss's law for magnetism), so its field lines have no start or end: they can *only* form closed loops, extend to infinity in both directions, or continue indefinitely without ever crossing itself.

Note that for this kind of drawing, where the field-line density is intended to be proportional to the field magnitude, it is important to represent all three dimensions. For example, consider the electric field arising from a single, isolated point charge. The electric field lines in this case are straight lines that emanate from the charge uniformly in all directions in three-dimensional space. This means that their density is proportional to $1/r^2$, the correct result consistent with Coulomb's law for this case. However, if the electric field lines for this setup were just drawn on a two-dimensional plane, their two-dimensional density would be proportional to $1/r$, an incorrect result for this situation.[1]

6.2   Examples

If the vector field describes a velocity field, then the field lines follow stream lines in the flow. Perhaps the most familiar example of a vector field described by field lines is the magnetic field, which is often depicted using field lines emanating from a magnet.

6.3   Divergence and curl

Field lines can be used to trace familiar quantities from vector calculus:

- Divergence may be easily seen through field lines, assuming the lines are drawn such that the density of field lines is proportional to the magnitude of the field (see above). In this case, the divergence may be seen as the beginning and ending of field lines. If the vector field is the resultant of radial inverse-square law fields with respect to one or more sources then this corresponds to the fact that the divergence of such a field is zero outside the sources. In a solenoidal vector field (i.e., a vector field where the divergence is zero everywhere), the field lines neither begin nor end; they either form closed loops, or go off to infinity in both directions. If a vector field has positive divergence in some area, there will be field lines starting from points in that area. If a vector field has negative divergence in some area, there will be field lines ending at points in that area.

- The Kelvin-Stokes theorem shows that field lines of a vector field with zero curl (i.e., a conservative vector field, e.g. a gravitational field or an electrostatic field) cannot be closed loops. In other words, curl is always

present when a field line forms a closed loop. It may be present in other situations too, such as a helical shape
of field lines.

6.4 Physical significance
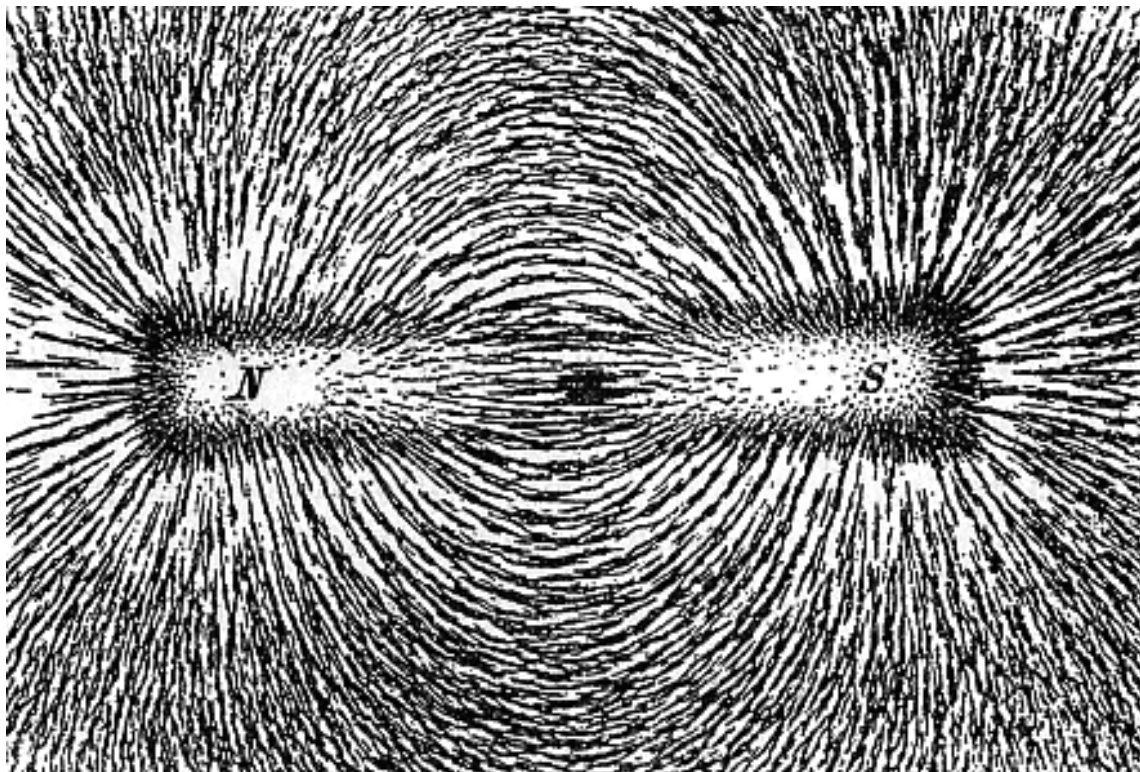
Iron filings arrange themselves so as to approximately depict some magnetic field lines. The magnetic field is created by a permanent magnet.

While field lines are a "mere" mathematical construction, in some circumstances they take on physical significance.
In fluid mechanics, the velocity field lines (streamlines) in steady flow represent the paths of particles of the fluid.
In the context of plasma physics, electrons or ions that happen to be on the same field line interact strongly, while
particles on different field lines in general do not interact. This is the same behavior that the particles of iron filings
exhibit in a magnetic field.

The iron filings in the photo appear to be aligning themselves with discrete field lines, but the situation is more
complex. It is easy to visualize as a two-stage-process: first, the filings are spread evenly over the magnetic field but
all aligned in the direction of the field. Then, based on the scale and ferromagnetic properties of the filings they
damp the field to either side, creating the apparent spaces between the lines that we see. Of course the two stages
described here happen concurrently until an equilibrium is achieved. Because the intrinsic magnetism of the filings
modifies the field, the lines shown by the filings are only an approximation of the field lines of the original magnetic
field. Magnetic fields are continuous, and do not have discrete lines.

6.5 See also

- Force field (physics)

- Field lines of Julia sets

- External ray — field lines of Douady-Hubbard potential of Mandelbrot set or filled-in Julia sets

- Line of force

- Vector field

6.6   References

- Griffiths, David J. (1998). *Introduction to Electrodynamics (3rd ed.)*. Prentice Hall. pp. 65–67 and 232. ISBN 0-13-805326-X.

- "Visualization of Fields and the Divergence and Curl" course notes from a course at the Massachusetts Institute of Technology.

[1] A. Wolf, S. J. Van Hook, E. R. Weeks, *Electric field line diagrams don't work* Am. J. Phys., Vol. 64, No. 6. (1996), pp. 714-724 DOI 10.1119/1.18237

6.7   External links

- Interactive Java applet showing the electric field lines of selected pairs of charges by Wolfgang Bauer

Chapter 7

Conservative vector field

In vector calculus a **conservative vector field** is a vector field that is the gradient of some function, known in this context as a scalar potential.[1] Conservative vector fields have the property that the line integral is path independent, i.e. the choice of integration path between any point and another does not change the result. Path independence of a line integral is equivalent to the vector field being conservative. A conservative vector field is also irrotational; in three dimensions this means that it has vanishing curl. An irrotational vector field is necessarily conservative provided that a certain condition on the geometry of the domain holds, i.e. the domain is simply connected.

Conservative vector fields appear naturally in mechanics: they are vector fields representing forces of physical systems in which energy is conserved.[2] For a conservative system, the work done in moving along a path in configuration space depends only on the endpoints of the path, so it is possible to define a potential energy independently of the path taken.

7.1 Informal treatment

In a two and three dimensional space, there is an ambiguity in taking an integral between two points as there are infinitely many paths between the two points - apart from the straight line formed between the two points one could choose a curved path of greater length as shown in the figure. Therefore in general the value of the integral depends on the path taken. However, in the special case of a conservative vector field, the value of the integral is independent of the path taken which can be thought of as a large-scale cancellation of all elements dR which don't have a component along the straight line between the two points. To visualise this, imagine two people climbing a cliff; one decides to scale the cliff by going vertically up it, and the second decides to walk along a winding path that is longer in length than the height of the cliff, but at only a small angle to the horizontal. Although the two hikers have taken different routes to get up to the top of the cliff, at the top they will have both gained the same amount of gravitational potential energy. This is because a gravitational field is conservative. As an example of a non-conservative field, imagine pushing a box from one end of a room to another. Pushing the box in a straight line across the room requires noticeably less work against friction than along a curved path covering a greater distance.

7.2 Intuitive explanation

M. C. Escher's painting *Ascending and Descending* illustrates a non-conservative vector field, impossibly made to appear to be the gradient of the varying height above ground as one moves along the staircase. It is "rotational" in that one can keep getting higher or keep getting lower while going around in circles. It is non-conservative in that one can return to one's starting point while ascending more than one descends or vice versa. On a real staircase the height above the ground is a scalar potential field: if one returns to the same place, one goes upward exactly as much as one goes downward. Its gradient would be a conservative vector field, and is irrotational. The situation depicted in the painting is impossible.
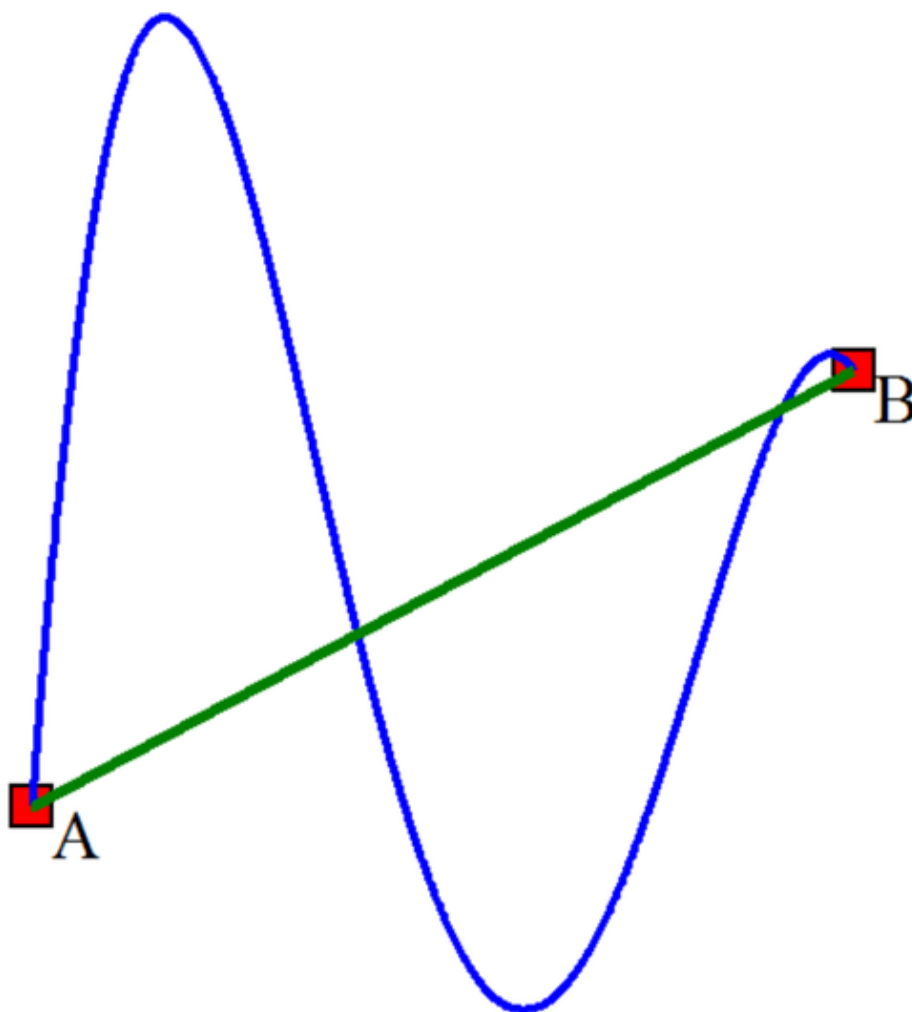
Depiction of two possible paths to integrate. In green in the simplest possible path, blue shows a more convoluted curve

7.3 Definition

A vector field $\mathbf{v}$ is said to be *conservative* if there exists a scalar field φ such that

$$\mathbf{v} = \nabla \varphi.$$

Here $\nabla \varphi$ denotes the gradient of φ . When the above equation holds, φ is called a scalar potential for $\mathbf{v}$.

The fundamental theorem of vector calculus states that any vector field can be expressed as the sum of a conservative vector field and a solenoidal field.

7.4 Path independence

Main article: Gradient theorem

A key property of a conservative vector field is that its integral along a path depends only on the endpoints of that path, not the particular route taken. Suppose that $S \subseteq \mathbb{R}^3$ is a region of three-dimensional space, and that P is a rectifiable path in S with start point A and end point B . If $\mathbf{v} = \nabla \varphi$ is a conservative vector field then the gradient theorem states that
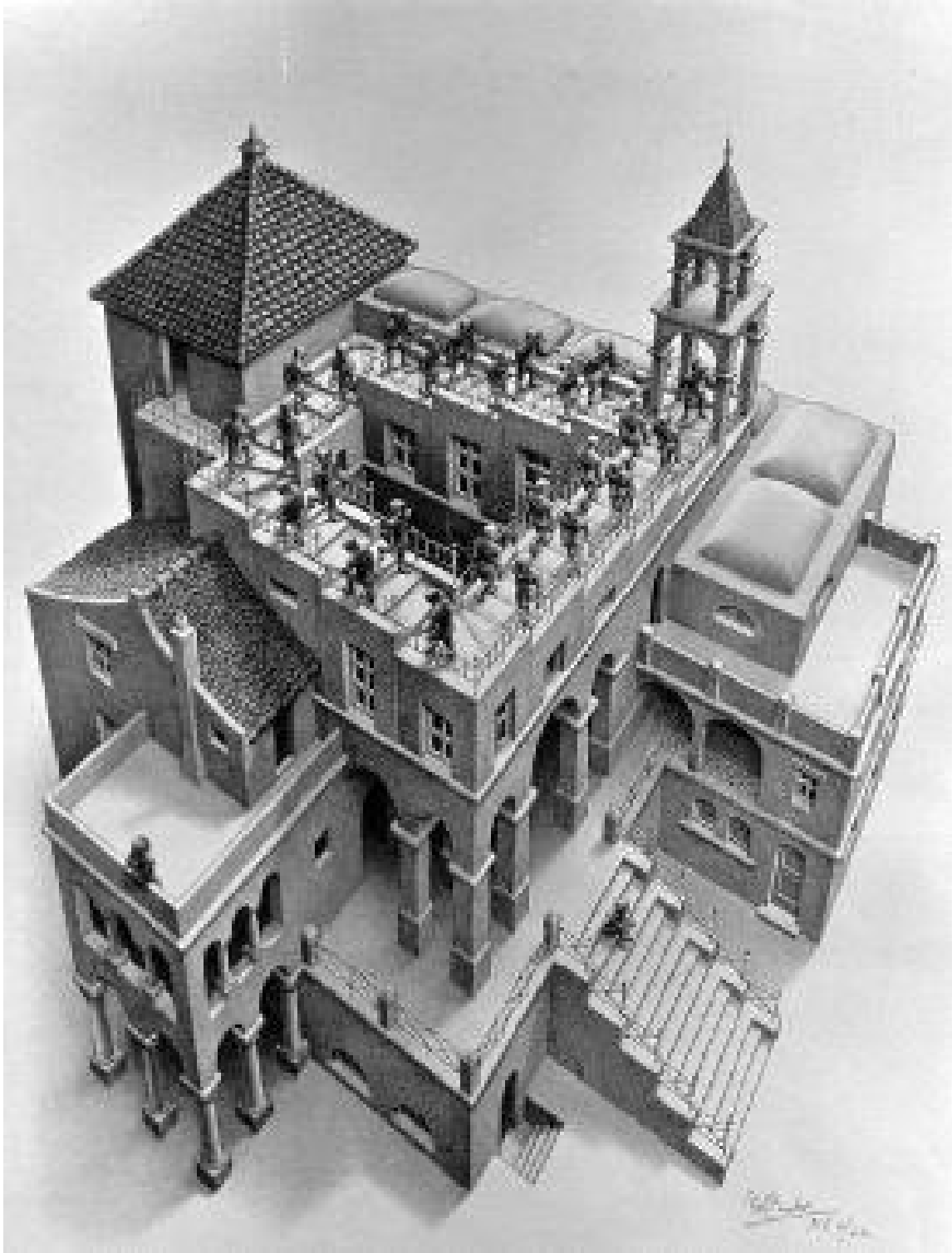
M. C. Escher's lithograph Ascending and Descending

$$\int_P \mathbf{v} \cdot d\mathbf{r} = \varphi(B) - \varphi(A).$$

This holds as a consequence of the chain rule and the fundamental theorem of calculus.

An equivalent formulation of this is to say that

$$\oint_C \mathbf{v} \cdot d\mathbf{r} = 0$$

for every closed loop C in *S*. The converse of this statement is also true: if the circulation of **v** around every closed loop in an open set *S* is zero, then **v** is a conservative vector field.

7.5   Irrotational vector fields
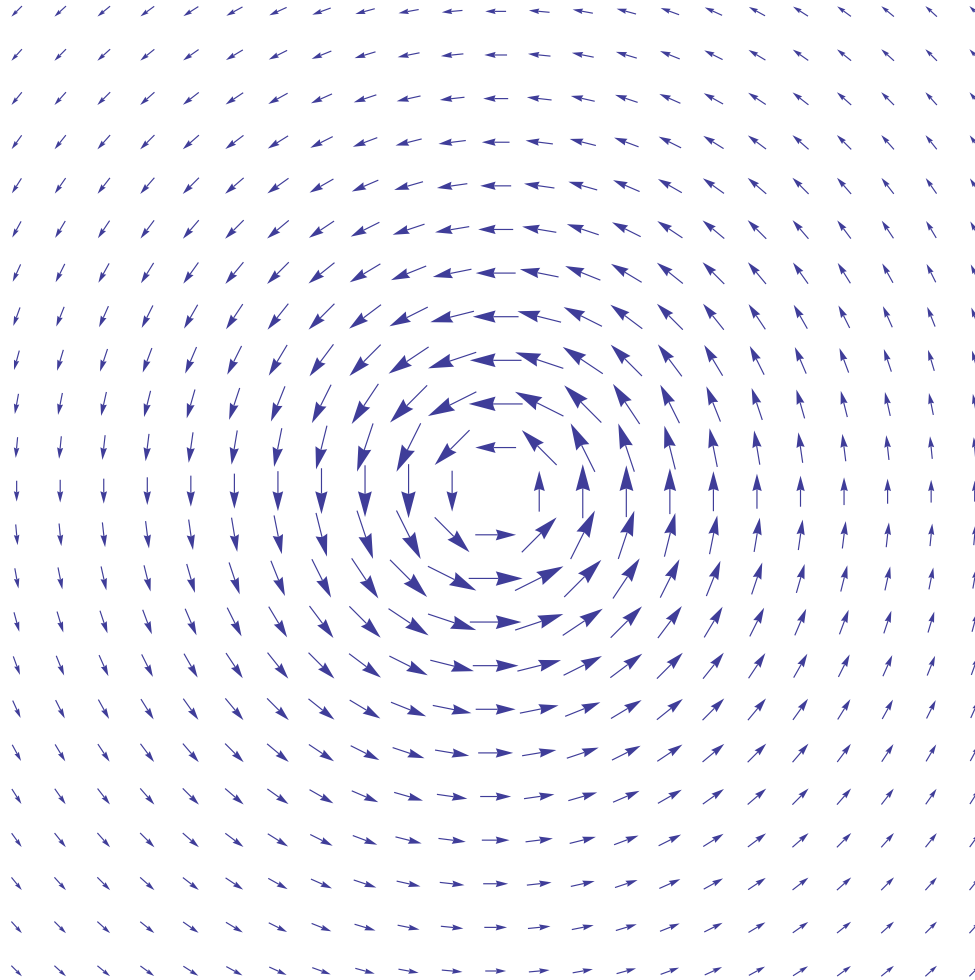
*The above field **v** = (−y/(x² + y²), +x/(x² + y²), 0) includes a vortex at its center, so it is* non-*irrotational; it is neither conservative, nor does it have path independence. However, any simply connected subset that excludes the vortex line (0,0,z) will have zero curl, ∇ × **v** = 0. Such vortex-free regions are examples of irrotational vector fields.*

A vector field **v** is said to be *irrotational* if its curl is zero. That is, if

$$\nabla \times \mathbf{v} = \mathbf{0}.$$

For this reason, such vector fields are sometimes referred to as **curl free field** (**curl-free vector field**) or *curl-less* vector fields.

It is an identity of vector calculus that for any scalar field φ :

$$\nabla \times \nabla \varphi = \mathbf{0}.$$

Therefore every conservative vector field is also an irrotational vector field.

Provided that S is a simply connected region, the converse of this is true: every irrotational vector field is also a conservative vector field.

The above statement is *not* true if S is not simply connected. Let S be the usual 3-dimensional space, except with the z -axis removed; that is $S = \mathbb{R}^3 \setminus \{(0, 0, z) \mid z \in \mathbb{R}\}$. Now define a vector field by

$$\mathbf{v} = \left(\frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2}, 0 \right).$$

Then $\mathbf{v}$ exists and has zero curl at every point in S ; that is $\mathbf{v}$ is irrotational. However the circulation of $\mathbf{v}$ around the unit circle in the x, y -plane is equal to 2π . Indeed we note that in polar coordinates $\mathbf{v} = \mathbf{e}_\phi / r$, so the integral over the unit circle is equal $\oint_C \mathbf{v} \cdot \mathbf{e}_\phi \mathrm{d}\phi = 2\pi$. Therefore $\mathbf{v}$ does not have the path independence property discussed above, and is not conservative. (However, in any simply connected subregion of S, it is still true that it is conservative. In fact, the field above is the gradient of $\arg(x + iy)$. As we know from complex analysis, this is a multi-valued function which requires a branch cut from the origin to infinity to be defined in a continuous way; hence, in a region that does not go around the z-axis, its gradient is conservative.)

In a simply connected region an irrotational vector field has the path independence property. This can be seen by noting that in such a region an irrotational vector field is conservative, and conservative vector fields have the path independence property. The result can also be proved directly by using Stokes' theorem. In a connected region any vector field which has the path independence property must also be irrotational.

More abstractly, a conservative vector field is an exact 1-form. That is, it is a 1-form equal to the exterior derivative of some 0-form (scalar field) ϕ . An irrotational vector field is a closed 1-form. Since $d^2 = 0$, any exact form is closed, so any conservative vector field is irrotational. The domain is simply connected if and only if its first homology group is 0, which is equivalent to its first cohomology group being 0. The first de Rham cohomology group H^1_{dR} is 0 if and only if all closed 1-forms are exact.

7.6 Irrotational flows

Main article: Vorticity

The flow velocity $\mathbf{v}$ of a fluid is a vector field, and the vorticity $\boldsymbol{\omega}$ of the flow can be defined by

$$\boldsymbol{\omega} = \nabla \times \mathbf{v}.$$

A common alternative notation for *vorticity* is ζ .[3]

If $\mathbf{v}$ is irrotational, with $\nabla \times \mathbf{v} = \mathbf{0}$, then the flow is said to be an **irrotational flow**. The vorticity of an irrotational flow is zero.[4]

Kelvin's circulation theorem states that a fluid that is irrotational in an inviscid flow will remain irrotational. This result can be derived from the vorticity transport equation, obtained by taking the curl of the Navier-stokes equations.

For a two-dimensional flow the vorticity acts as a measure of the *local* rotation of fluid elements. Note that the vorticity does *not* imply anything about the global behaviour of a fluid. It is possible for a fluid traveling in a straight line to have vorticity, and it is possible for a fluid which moves in a circle to be irrotational.

7.7 Conservative forces

If the vector field associated to a force $\mathbf{F}$ is conservative then the force is said to be a conservative force.

The most prominent examples of conservative forces are the force of gravity and the electric field associated to a static charge. According to Newton's law of gravitation, the gravitational force, $\mathbf{F}_G$, acting on a mass m , due to a mass M which is a distance r away, obeys the equation
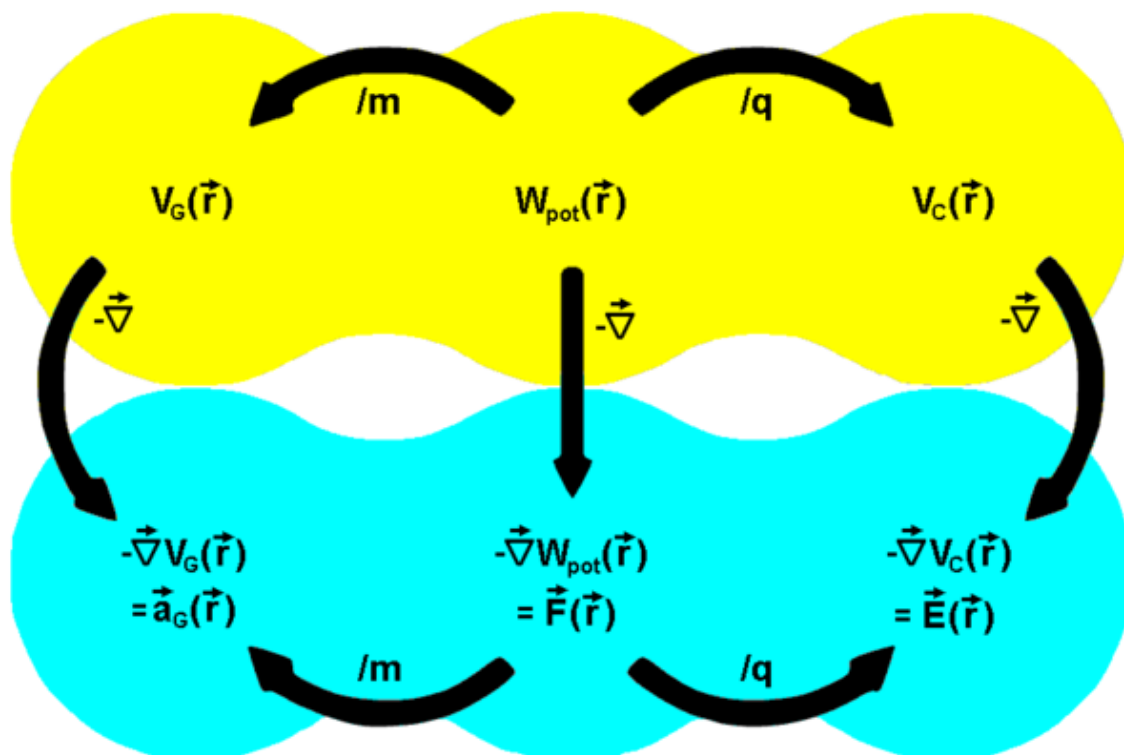
Examples of potential and gradient fields in physics
Scalar fields (scalar potentials) (yellow)*: **VG** - gravitational potential; **Wpot** - potential energy; **VC** - Coulomb potential;* Vector
fields (gradient fields) (cyan)*: **aG** - gravitational acceleration; **F** - force; **E** - electric field strength*

$$\mathbf{F}_G = -\frac{GmM\hat{\mathbf{r}}}{r^2},$$

where G is the gravitational constant and $\hat{\mathbf{r}}$ is a unit vector pointing from M towards m . The force of gravity is
conservative because $\mathbf{F}_G = -\nabla\Phi_G$, where

$$\Phi_G = -\frac{GmM}{r}$$

is the gravitational potential energy.

For conservative forces, *path independence* can be interpreted to mean that the work done in going from a point A to
a point B is independent of the path chosen, and that the work W done in going around a closed loop is zero:

$$W = \oint_C \mathbf{F} \cdot d\mathbf{r} = 0.$$

The total energy of a particle moving under the influence of conservative forces is conserved, in the sense that a loss
of potential energy is converted to an equal quantity of kinetic energy or vice versa.

7.8   See also

- Beltrami vector field
- Complex lamellar vector field
- Helmholtz decomposition

- Laplacian vector field

- Longitudinal and transverse vector fields

- Potential field

- Solenoidal vector field

7.9 Citations and sources

Citations

[1] Marsden, Jerrold; Tromba, Anthony (2003). *Vector calculus* (Fifth ed.). W.H.Freedman and Company. pp. 550–561.

[2] George B. Arfken and Hans J. Weber, *Mathematical Methods for Physicists*, 6th edition, Elsevier Academic Press (2005)

[3] Clancy, L.J., *Aerodynamics*, Section 7.11, Pitman Publishing Limited, London. ISBN 0-273-01120-0

[4] Liepmann, H.W.; Roshko, A. (1993) [1957], *Elements of Gas Dynamics*, Courier Dover Publications, ISBN 0-486-41963-0, pp. 194–196.

Sources

- D. J. Acheson, *Elementary Fluid Dynamics*, Oxford University Press (2005)

Chapter 8

Vector field
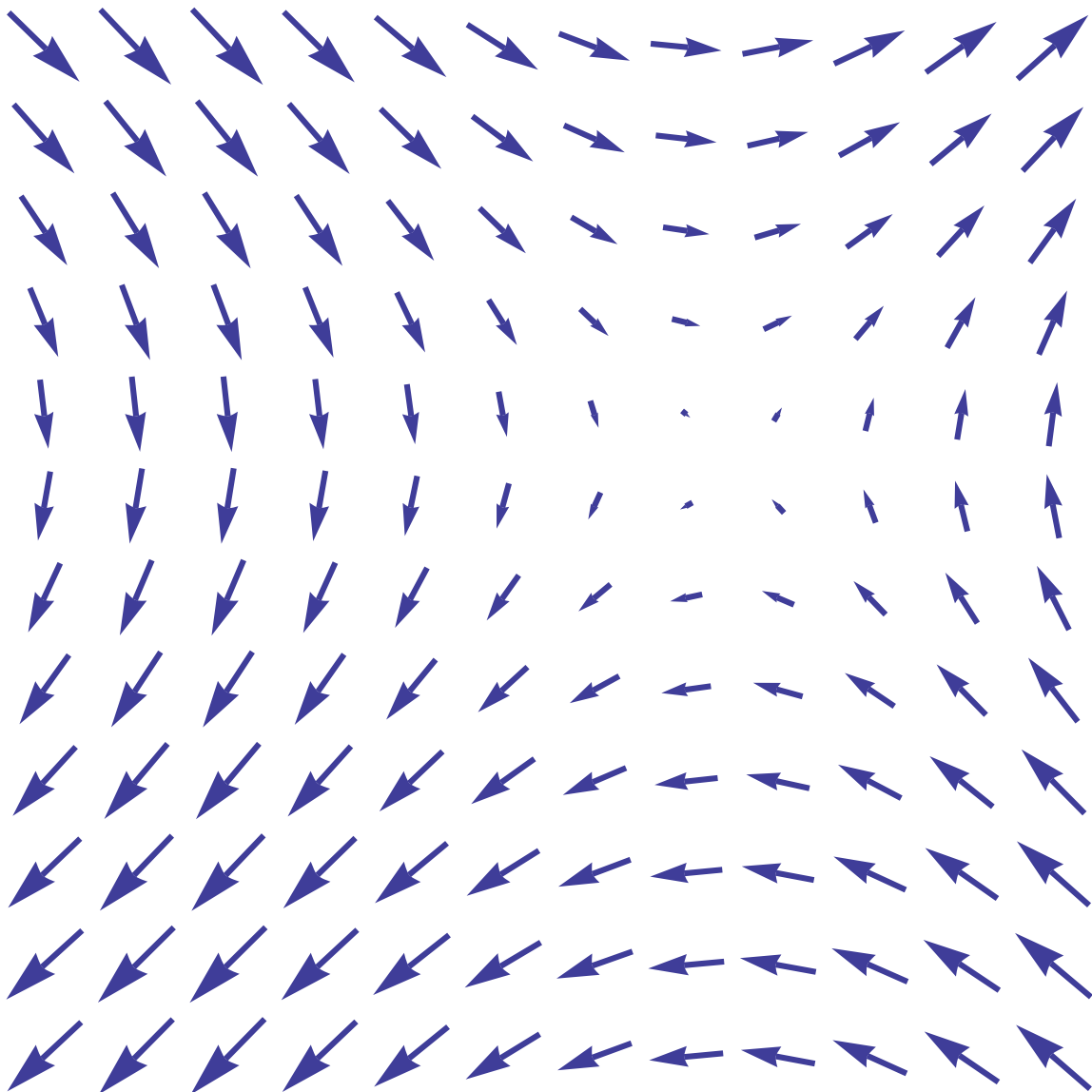
A portion of the vector field (sin y, *sin* x*)*

In vector calculus, a **vector field** is an assignment of a vector to each point in a subset of space.[1] A vector field in the plane, for instance, can be visualized as a collection of arrows with a given magnitude and direction each attached to a point in the plane. Vector fields are often used to model, for example, the speed and direction of a moving

fluid throughout space, or the strength and direction of some force, such as the magnetic or gravitational force, as it changes from point to point.

The elements of differential and integral calculus extend to vector fields in a natural way. When a vector field represents force, the line integral of a vector field represents the work done by a force moving along a path, and under this interpretation conservation of energy is exhibited as a special case of the fundamental theorem of calculus. Vector fields can usefully be thought of as representing the velocity of a moving flow in space, and this physical intuition leads to notions such as the divergence (which represents the rate of change of volume of a flow) and curl (which represents the rotation of a flow).

In coordinates, a vector field on a domain in n-dimensional Euclidean space can be represented as a vector-valued function that associates an n-tuple of real numbers to each point of the domain. This representation of a vector field depends on the coordinate system, and there is a well-defined transformation law in passing from one coordinate system to the other. Vector fields are often discussed on open subsets of Euclidean space, but also make sense on other subsets such as surfaces, where they associate an arrow tangent to the surface at each point (a tangent vector).

More generally, vector fields are defined on differentiable manifolds, which are spaces that look like Euclidean space on small scales, but may have more complicated structure on larger scales. In this setting, a vector field gives a tangent vector at each point of the manifold (that is, a section of the tangent bundle to the manifold). Vector fields are one kind of tensor field.

8.1 Definition

8.1.1 Vector fields on subsets of Euclidean space
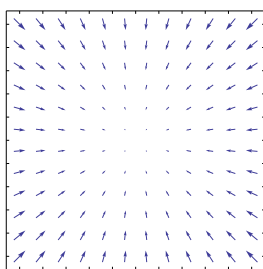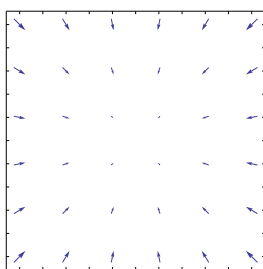
Two representations of the same vector field: $\mathbf{v}(x, y) = -\mathbf{r}$. The arrows depict the field at discrete points, however, the field exists everywhere.

Given a subset S in $\mathbf{R}^n$, a **vector field** is represented by a vector-valued function $V\colon S \to \mathbf{R}^n$ in standard Cartesian coordinates $(x_1, ..., xn)$. If each component of V is continuous, then V is a continuous vector field, and more generally V is a C^k vector field if each component of V is k times continuously differentiable.

A vector field can be visualized as assigning a vector to individual points within an n-dimensional space.[1]

Given two C^k-vector fields V, W defined on S and a real valued C^k-function f defined on S, the two operations scalar multiplication and vector addition

$$(fV)(p) := f(p)V(p)$$

$$(V + W)(p) := V(p) + W(p)$$

define the module of C^k-vector fields over the ring of C^k-functions.

8.1.2   Coordinate transformation law

In physics, a vector is additionally distinguished by how its coordinates change when one measures the same vector with respect to a different background coordinate system. The transformation properties of vectors distinguish a vector as a geometrically distinct entity from a simple list of scalars, or from a covector.

Thus, suppose that $(x_1,...,xn)$ is a choice of Cartesian coordinates, in terms of which the components of the vector V are

$$V_x = (V_{1,x}, \ldots, V_{n,x})$$

and suppose that $(y_1,...,yn)$ are n functions of the xi defining a different coordinate system. Then the components of the vector V in the new coordinates are required to satisfy the transformation law

Such a transformation law is called contravariant. A similar transformation law characterizes vector fields in physics: specifically, a vector field is a specification of n functions in each coordinate system subject to the transformation law (**1**) relating the different coordinate systems.

Vector fields are thus contrasted with scalar fields, which associate a number or *scalar* to every point in space, and are also contrasted with simple lists of scalar fields, which do not transform under coordinate changes.

8.1.3   Vector fields on manifolds

Given a differentiable manifold M, a **vector field** on M is an assignment of a tangent vector to each point in M.[2] More precisely, a vector field F is a mapping from M into the tangent bundle TM so that $p \circ F$ is the identity mapping where p denotes the projection from TM to M. In other words, a vector field is a section of the tangent bundle.

If the manifold M is smooth or analytic—that is, the change of coordinates is smooth (analytic)—then one can make sense of the notion of smooth (analytic) vector fields. The collection of all smooth vector fields on a smooth manifold M is often denoted by $\Gamma(TM)$ or $C^\infty(M,TM)$ (especially when thinking of vector fields as sections); the collection of all smooth vector fields is also denoted by $\mathfrak{x}(M)$ (a fraktur "X").

8.2   Examples

- A vector field for the movement of air on Earth will associate for every point on the surface of the Earth a vector with the wind speed and direction for that point. This can be drawn using arrows to represent the wind; the length (magnitude) of the arrow will be an indication of the wind speed. A "high" on the usual barometric pressure map would then act as a source (arrows pointing away), and a "low" would be a sink (arrows pointing towards), since air tends to move from high pressure areas to low pressure areas.

- Velocity field of a moving fluid. In this case, a velocity vector is associated to each point in the fluid.

- Streamlines, Streaklines and Pathlines are 3 types of lines that can be made from vector fields. They are :

     streaklines — as revealed in wind tunnels using smoke.
     streamlines (or fieldlines)— as a line depicting the instantaneous field at a given time.
     pathlines — showing the path that a given particle (of zero mass) would follow.

- Magnetic fields. The fieldlines can be revealed using small iron filings.

- Maxwell's equations allow us to use a given set of initial conditions to deduce, for every point in Euclidean space, a magnitude and direction for the force experienced by a charged test particle at that point; the resulting vector field is the electromagnetic field.

- A gravitational field generated by any massive object is also a vector field. For example, the gravitational field vectors for a spherically symmetric body would all point towards the sphere's center with the magnitude of the vectors reducing as radial distance from the body increases.
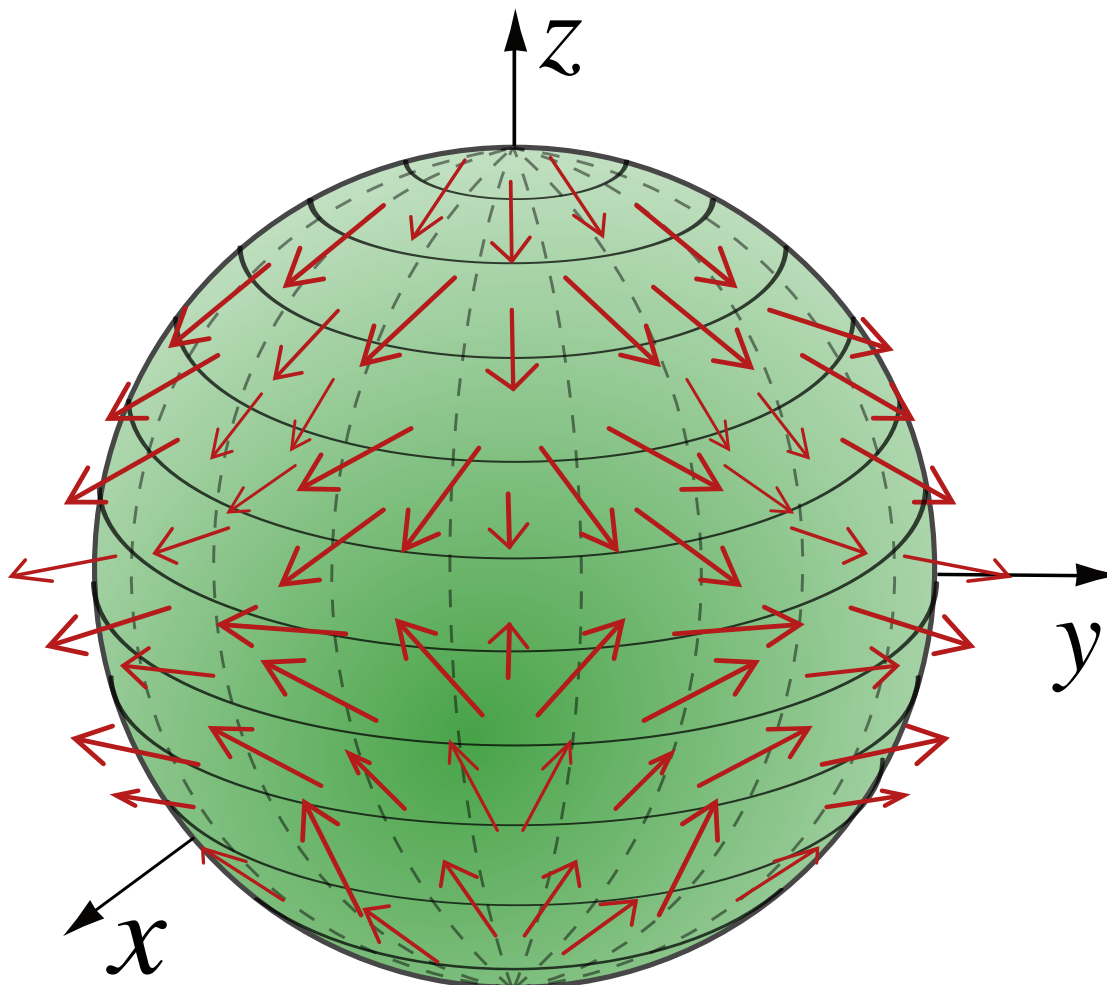
A vector field on a sphere

8.2.1 Gradient field

Vector fields can be constructed out of scalar fields using the gradient operator (denoted by the del: ∇).[3]

A vector field V defined on a set S is called a **gradient field** or a **conservative field** if there exists a real-valued function (a scalar field) f on S such that

$$V = \nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial x_3}, \ldots, \frac{\partial f}{\partial x_n} \right).$$

The associated flow is called the **gradient flow**, and is used in the method of gradient descent.

The path integral along any closed curve γ ($\gamma(0) = \gamma(1)$) in a conservative field is zero:

$$\oint_\gamma \langle V(x), \mathrm{d}x \rangle = \oint_\gamma \langle \nabla f(x), \mathrm{d}x \rangle = f(\gamma(1)) - f(\gamma(0)).$$

where the angular brackets and comma: $\langle , \rangle$ denotes the inner product of two vectors (strictly speaking – the integrand $V(x)$ is a 1-form rather than a vector in the elementary sense).[4]

8.2.2 Central field

A C^∞-vector field over $\mathbf{R}^n \setminus \{0\}$ is called a **central field** if
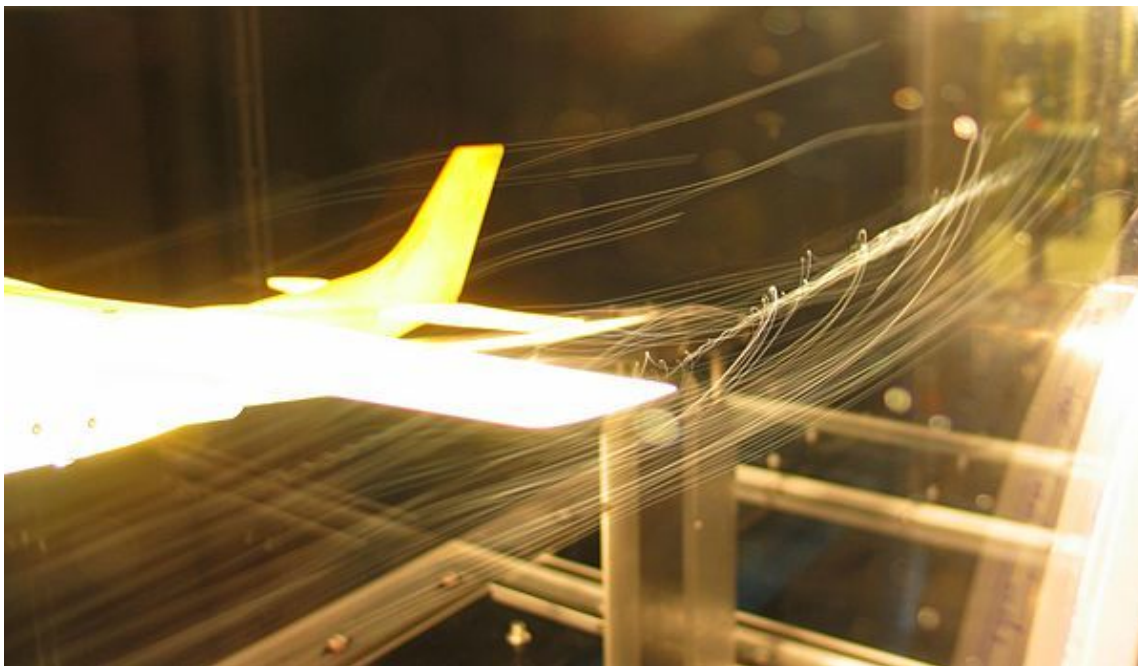
The flow field around an airplane is a vector field in $\mathbf{R}^3$, here visualized by bubbles that follow the streamlines showing a wingtip vortex.

$$V(T(p)) = T(V(p)) \qquad (T \in \mathbf{O}(n, \mathbf{R}))$$

where $\mathrm{O}(n, \mathbf{R})$ is the orthogonal group. We say central fields are invariant under orthogonal transformations around 0.

The point 0 is called the **center** of the field.

Since orthogonal transformations are actually rotations and reflections, the invariance conditions mean that vectors of a central field are always directed towards, or away from, 0; this is an alternate (and simpler) definition. A central field is always a gradient field, since defining it on one semiaxis and integrating gives an antigradient.

8.3   Operations on vector fields

8.3.1   Line integral

Main article: Line integral

A common technique in physics is to integrate a vector field along a curve, i.e. to determine its line integral. Given a particle in a gravitational vector field, where each vector represents the force acting on the particle at a given point in space, the line integral is the work done on the particle when it travels along a certain path.

The line integral is constructed analogously to the Riemann integral and it exists if the curve is rectifiable (has finite length) and the vector field is continuous.

Given a vector field V and a curve γ parametrized by $[a, b]$ (where a and b are real) the line integral is defined as

$$\int_\gamma \langle V(x), \mathrm{d}x \rangle = \int_a^b \langle V(\gamma(t)), \gamma'(t) \, \mathrm{d}t \rangle.$$
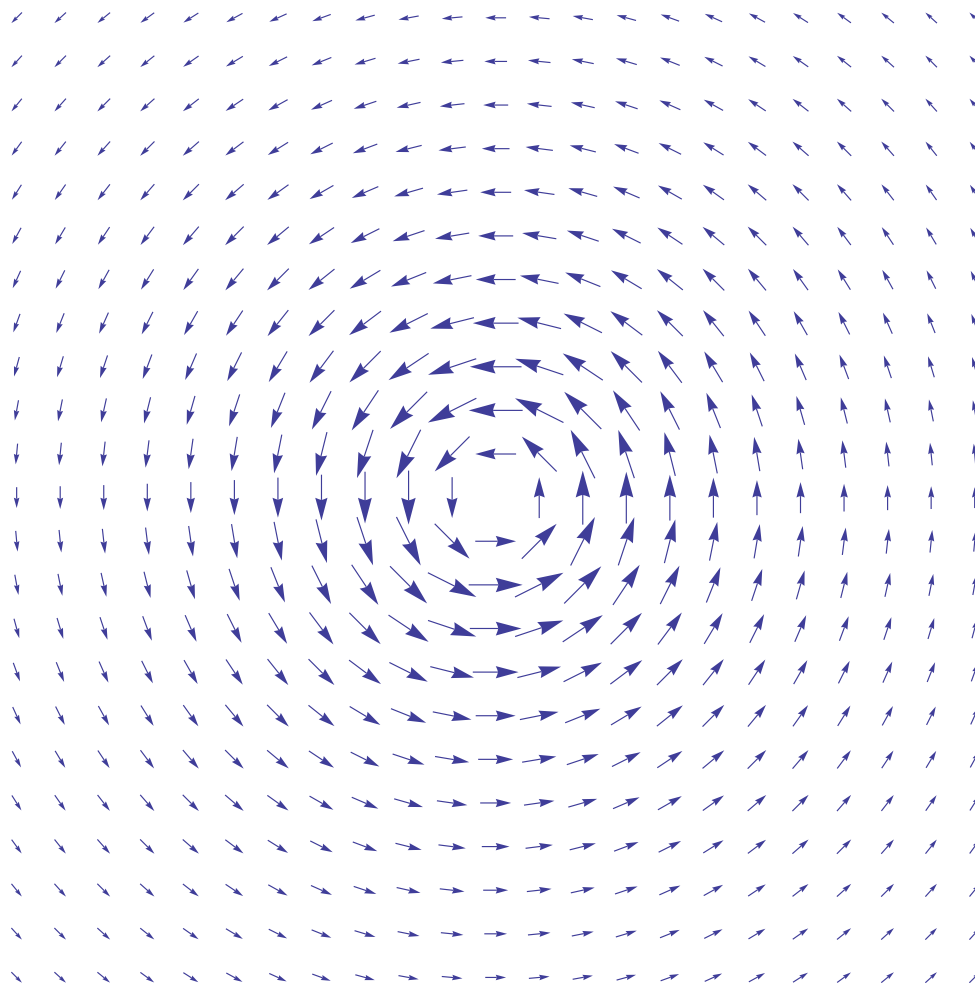
A vector field that has circulation about a point cannot be written as the gradient of a function.

8.3.2 Divergence

Main article: Divergence

The divergence of a vector field on Euclidean space is a function (or scalar field). In three-dimensions, the divergence is defined by

$$\operatorname{div}\mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z},$$

with the obvious generalization to arbitrary dimensions. The divergence at a point represents the degree to which a small volume around the point is a source or a sink for the vector flow, a result which is made precise by the divergence theorem.

The divergence can also be defined on a Riemannian manifold, that is, a manifold with a Riemannian metric that measures the length of vectors.

8.3.3 Curl

Main article: Curl (mathematics)

The curl is an operation which takes a vector field and produces another vector field. The curl is defined only in

three-dimensions, but some properties of the curl can be captured in higher dimensions with the exterior derivative. In three-dimensions, it is defined by

$$\text{curl } \mathbf{F} = \nabla \times \mathbf{F} = \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z} \right) \mathbf{e}_1 - \left(\frac{\partial F_3}{\partial x} - \frac{\partial F_1}{\partial z} \right) \mathbf{e}_2 + \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) \mathbf{e}_3.$$

The curl measures the density of the angular momentum of the vector flow at a point, that is, the amount to which the flow circulates around a fixed axis. This intuitive description is made precise by Stokes' theorem.

8.3.4   Index of a vector field

The index of a vector field is a way of describing the behaviour of a vector field around an isolated zero (i.e. non-singular point) which can distinguish saddles from sources and sinks. Take a small sphere around the zero so that no other zeros are included. A map from this sphere to a unit sphere of dimensions $n - 1$ can be constructed by dividing each vector by its length to form a unit length vector which can then be mapped to the unit sphere. The index of the vector field at the point is the degree of this map. The index of the vector field is the sum of the indices of each zero.

The index will be zero around any non singular point, it is +1 around sources and sinks and −1 around saddles. In two dimensions the index is equivalent to the winding number. For an ordinary sphere in three dimension space it can be shown that the index of any vector field on the sphere must be two, this leads to the hairy ball theorem which shows that every such vector field must have a zero. This theorem generalises to the Poincaré–Hopf theorem which relates the index to the Euler characteristic of the space.

8.4   History
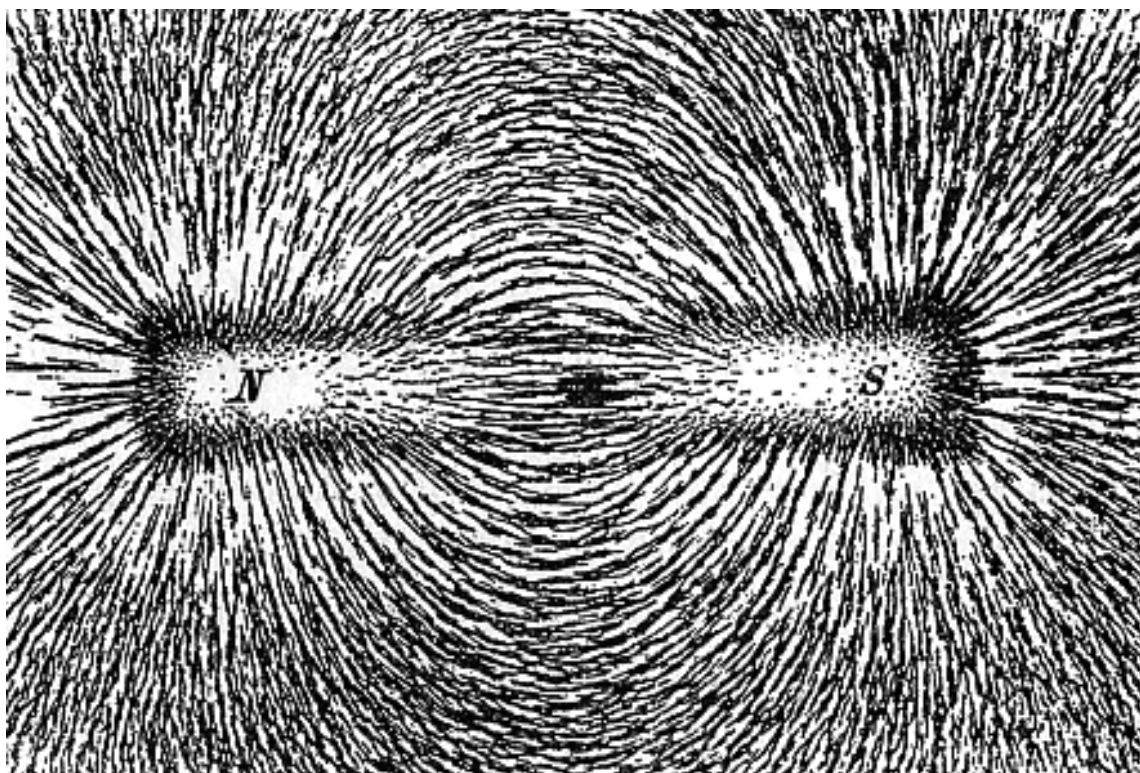
Magnetic field lines of an iron bar (magnetic dipole)

Vector fields arose originally in classical field theory in 19th century physics, specifically in magnetism. They were formalized by Michael Faraday, in his concept of *lines of force,* who emphasized that the field *itself* should be an object of study, which it has become throughout physics in the form of field theory.

In addition to the magnetic field, other phenomena that were modeled as vector fields by Faraday include the electrical field and light field.

8.5 Flow curves

Main article: Integral curve

Consider the flow of a fluid through a region of space. At any given time, any point of the fluid has a particular velocity associated with it; thus there is a vector field associated to any flow. The converse is also true: it is possible to associate a flow to a vector field having that vector field as its velocity.

Given a vector field V defined on S, one defines curves $\gamma(t)$ on S such that for each t in an interval I

$$\gamma'(t) = V(\gamma(t)).$$

By the Picard–Lindelöf theorem, if V is Lipschitz continuous there is a *unique* C^1-curve γ_x for each point x in S so that

$$\gamma_x(0) = x$$

$$\gamma_x'(t) = V(\gamma_x(t)) \qquad (t \in (-\varepsilon, +\varepsilon) \subset \mathbf{R}).$$

The curves γ_x are called **flow curves** of the vector field V and partition S into equivalence classes. It is not always possible to extend the interval $(-\varepsilon, +\varepsilon)$ to the whole real number line. The flow may for example reach the edge of S in a finite time. In two or three dimensions one can visualize the vector field as giving rise to a flow on S. If we drop a particle into this flow at a point p it will move along the curve γ_p in the flow depending on the initial point p. If p is a stationary point of V then the particle will remain at p.

Typical applications are streamline in fluid, geodesic flow, and one-parameter subgroups and the exponential map in Lie groups.

8.5.1 Complete vector fields

A vector field is **complete** if its flow curves exist for all time.[5] In particular, compactly supported vector fields on a manifold are complete. If X is a complete vector field on M, then the one-parameter group of diffeomorphisms generated by the flow along X exists for all time.

8.6 Difference between scalar and vector field

The difference between a scalar and vector field is not that a scalar is just one number while a vector is several numbers. The difference is in how their coordinates respond to coordinate transformations. A scalar *is* a coordinate whereas a vector *can be described* by coordinates, but it *is not* the collection of its coordinates.

8.6.1 Example 1

This example is about 2-dimensional Euclidean space ($\mathbf{R}^2$) where we examine Euclidean (x, y) and polar (r, θ) coordinates (which are undefined at the origin). Thus $x = r \cos \theta$ and $y = r \sin \theta$ and also $r^2 = x^2 + y^2$, $\cos \theta = x/(x^2 + y^2)^{1/2}$ and $\sin \theta = y/(x^2 + y^2)^{1/2}$. Suppose we have a scalar field which is given by the constant function 1, and a vector field which attaches a vector in the r-direction with length 1 to each point. More precisely, they are given by the functions

$$s_{\text{polar}} : (r, \theta) \mapsto 1, \quad v_{\text{polar}} : (r, \theta) \mapsto (1, 0).$$

Let us convert these fields to Euclidean coordinates. The vector of length 1 in the r-direction has the x coordinate $\cos\theta$ and the y coordinate $\sin\theta$. Thus in Euclidean coordinates the same fields are described by the functions

$$s_{\text{Euclidean}} : (x, y) \mapsto 1,$$

$$v_{\text{Euclidean}} : (x, y) \mapsto (\cos\theta, \sin\theta) = \left(\frac{x}{\sqrt{x^2 + y^2}}, \frac{y}{\sqrt{x^2 + y^2}} \right).$$

We see that while the scalar field remains the same, the vector field now looks different. The same holds even in the 1-dimensional case, as illustrated by the next example.

8.6.2 Example 2

Consider the 1-dimensional Euclidean space **R** with its standard Euclidean coordinate x. Suppose we have a scalar field and a vector field which are both given in the x coordinate by the constant function 1,

$$s_{\text{Euclidean}} : x \mapsto 1, \quad v_{\text{Euclidean}} : x \mapsto 1.$$

Thus, we have a scalar field which has the value 1 everywhere and a vector field which attaches a vector in the x-direction with magnitude 1 unit of x to each point.

Now consider the coordinate $\xi := 2x$. If x changes one unit then ξ changes 2 units. But since we wish the integral of v along a path to be independent of coordinate, this means v*dx=v'*dξ. So from x increase by 1 unit, ξ increases by 1/2 unit, so v' must be 2. Thus this vector field has a magnitude of 2 in units of ξ. Therefore, in the ξ coordinate the scalar field and the vector field are described by the functions

$$s_{\text{unusual}} : \xi \mapsto 1, \quad v_{\text{unusual}} : \xi \mapsto 2$$

which are different.

8.7 f-relatedness

Given a smooth function between manifolds, $f \colon M \to N$, the derivative is an induced map on tangent bundles, $f^* \colon TM \to TN$. Given vector fields $V \colon M \to TM$ and $W \colon N \to TN$, we say that W is f-related to V if the equation $W \circ f^* = f^* \circ V$ holds.

If V_i is f-related to W_i, $i = 1, 2$, then the Lie bracket $[V_1, V_2]$ is f-related to $[W_1, W_2]$.

8.8 Generalizations

Replacing vectors by p-vectors (pth exterior power of vectors) yields p-vector fields; taking the dual space and exterior powers yields differential k-forms, and combining these yields general tensor fields.

Algebraically, vector fields can be characterized as derivations of the algebra of smooth functions on the manifold, which leads to defining a vector field on a commutative algebra as a derivation on the algebra, which is developed in the theory of differential calculus over commutative algebras.

8.9 See also

- Eisenbud–Levine–Khimshiashvili signature formula
- Field line

- Field strength

- Lie derivative

- Scalar field

- Time-dependent vector field

- Vector fields in cylindrical and spherical coordinates

- Tensor fields

8.10 References

[1] Galbis, Antonio & Maestre, Manuel (2012). *Vector Analysis Versus Vector Calculus*. Springer. p. 12. ISBN 978-1-4614-2199-3.

[2] Tu, Loring W. (2010). "Vector fields". *An Introduction to Manifolds*. Springer. p. 149. ISBN 978-1-4419-7399-3.

[3] Dawber, P.G. (1987). *Vectors and Vector Operators*. CRC Press. p. 29. ISBN 978-0-85274-585-4.

[4] T. Frankel (2012), *The Geometry of Physics* (3rd ed.), Cambridge University Press, p. xxxviii, ISBN 978-1107-602601

[5] Sharpe, R. (1997). *Differential geometry*. Springer-Verlag. ISBN 0-387-94732-9.

8.11 Bibliography

- Hubbard, J. H.; Hubbard, B. B. (1999). *Vector calculus, linear algebra, and differential forms. A unified approach*. Upper Saddle River, NJ: Prentice Hall. ISBN 0-13-657446-7.

- Warner, Frank (1983) [1971]. *Foundations of differentiable manifolds and Lie groups*. New York-Berlin: Springer-Verlag. ISBN 0-387-90894-3.

- Boothby, William (1986). *An introduction to differentiable manifolds and Riemannian geometry*. Pure and Applied Mathematics, volume 120 (second ed.). Orlando, FL: Academic Press. ISBN 0-12-116053-X.

8.12 External links

- Hazewinkel, Michiel, ed. (2001), "Vector field", *Encyclopedia of Mathematics*, Springer, ISBN 978-1-55608-010-4

- Vector field — Mathworld

- Vector field — PlanetMath

- 3D Magnetic field viewer

- Vector fields and field lines

- Vector field simulation An interactive application to show the effects of vector fields

- Vector Fields Software 2d & 3d electromagnetic design software that can be used to visualise vector fields and field lines

8.13   Text and image sources, contributors, and licenses

8.13.1   Text

- **Electric charge** *Source:* https://en.wikipedia.org/wiki/Electric_charge?oldid=680302226 *Contributors:* AxelBoldt, Mav, Andre Engels, Roadrunner, Peterlin~enwiki, Heron, JohnOwens, Michael Hardy, Ixfd64, Delirium, Looxix~enwiki, Ellywa, Mdebets, Glenn, Rossami, Nikai, Andres, Raven in Orbit, Reddi, Omegatron, Gakrivas, Lumos3, Rogper~enwiki, Gentgeen, Robbot, Fredrik, Dukeofomnium, Wikibot, Fuelbottle, Wjbeaty, Giftlite, DavidCary, Herbee, Snowdog, Dratman, Valen~enwiki, RScheiber, Jason Quinn, Brockert, OldakQuill, Manuel Anastácio, LiDaobing, Karol Langner, Icairns, Iantresman, GNU, Vincom2, Discospinster, Guanabot, Jpk, Dbachmann, ZeroOne, Laurascudder, Bobo192, Rbj, Giraffedata, Kjkolb, Scentoni, Mdd, Alansohn, Atlant, ABCD, Velella, Wtshymanski, HenkvD, Mikeo, DV8 2XL, Gene Nygaard, HenryLi, Oleg Alexandrov, Nuno Tavares, Cimex, Rocastelo, StradivariusTV, Oliphaunt, BillC, Eleassar777, Cyberman, Palica, BD2412, Demonuk, Edison, SMC, Krash, Dougluce, FlaBot, Psyphen, Nivix, Alfred Centauri, Gurch, Kri, Gdrbot, Manscher, YurikBot, Bambaiah, Lucinos~enwiki, Stephenb, Manop, Pseudomonas, JDoorjam, TDogg310, Chichui, Kkmurray, Wknight94, Light current, Enormousdude, Johndburger, Tcsetattr, Pinikas, Reyk, Canley, Geoffrey.landis, JDspeeder1, GrinBot~enwiki, Mejor Los Indios, Sbyrnes321, Marquez~enwiki, Moeron, Vald, Thunderboltz, Dmitry sychov, HalfShadow, Gilliam, Oscarthecat, Andy M. Wang, Chris the speller, Lenko, DHN-bot~enwiki, Dual Freq, Hallenrm, Rrburke, The tooth, MichaelBillington, Hgilbert, Drphilharmonic, Daniel.Cardenas, Springnuts, Yevgeny Kats, Andrei Stroe, DJIndica, Naui~enwiki, Nmnogueira, SashatoBot, Richard L. Peterson, Slowmover, Cronholm144, Mgiganteus1, Bjankuloski06en~enwiki, Nonsuch, Ben Moore, RandomCritic, MarkSutton, Stikonas, Dicklyon, Levineps, Igoldste, Tawkerbot2, Chetvorno, JForget, CmdrObot, Kehrli, Jsd, Myasuda, Cydebot, Fl, Bvcrist, Meno25, Gogo Dodo, WISo, Christian75, Ssilvers, Thijs!bot, Epbr123, Barticus88, N5iln, Mojo Hand, Headbomb, Gerry Ashton, Escarbot, Aadal, AntiVandalBot, Seaphoto, Prolog, DarkAudit, Lyricmac, Tim Shuba, WikifingHelper, Asgrrr, JAnDbot, Acroterion, Bongwarrior, VoABot II, J2thawiki, Sstolper, Jjurik, Bubba hotep, User A1, DerHexer, InvertRect, Robin S, MartinBot, M. Bilal Shafiq, LedgendGamer, Pharaoh of the Wizards, Numbo3, Hans Dunkelberg, NightFalcon90909, Uncle Dick, Ginsengbomb, Katalaveno, DarkFalls, NewEnglandYankee, QuickClown, Juliancolton, ACBest, Treisijs, Lseixas, Jefferson Anderson, Sheliak, Philip Trueman, TXiKiBoT, The Original Wildbear, Ayan2289, Nickipedia 008, LuizBalloti, Monty845, Jpalpant, Biscuittin, Demmy100, SieBot, Gerakibot, Caltas, Gastin, Wing gundam, Msadaghd, JerrySteal, Jojalozzo, Oxymoron83, Faradayplank, Avnjay, Anchor Link Bot, Neo., Loren.wilton, ClueBot, The Thing That Should Not Be, Arakunem, Termine, Mild Bill Hiccup, Stephaninator, LeoFrank, Excirial, Kocher2006, Jusdafax, Brews ohare, Cenarium, Jotterbot, PhySusie, SchreiberBike, Wuzur, JDPhD, Versus22, Thinking Stone, Rror, Cernms, Truthnlove, Addbot, Some jerk on the Internet, CanadianLinuxUser, NjardarBot, LaaknorBot, Scottyferguson, LinkFA-Bot, Naidevinci, Ocwaldron, Tide rolls, Lightbot, JDSperling, Legobot, Luckas-bot, Yobot, CinchBug, Duping Man, AnomieBOT, DemocraticLuntz, Sertion, Jim1138, IRP, Pyrrhus16, Kingpin13, Bluerasberry, Materialscientist, Geek1337~enwiki, ImperatorExercitus, Xqbot, TheAMmollusc, Phazvmk, Addihockey10, Capricorn42, Nnivi, ProtectionTaggingBot, RibotBOT, Srr712, A. di M., Constructive editor, Frozenevolution, Ryryrules100, Jc3s5h, Drunauthorized, Mithrandir, Steve Quinn, Davidteng, Fast kartwheels, BenzolBot, DivineAlpha, AstaBOTh15, Pinethicket, Jivee Blau, Calmer Waters, Tinton5, MastiBot, Serols, Meaghan, Lalrang2007, Logical Gentleman, FoxBot, TobeBot, SchreyP, Jonkerz, Ndkartik, Vrenator, Taytaylisious09, Ammodramus, Jamietw, DARTH SIDIOUS 2, Eshmate, Irfanyousufdar, EmausBot, John of Reading, GoingBatty, K6ka, Darkfight, Hhhippo, JSquish, Harddk, Stephen C Wells, Liam McM, Sonygal, L Kensington, Donner60, Peter Karlsen, Sven Manguard, Planetscared, ClueBot NG, Jack Greenmaven, Cking1414, Ihwood, Ulflund, CocuBot, MelbourneStar, O.Koslowski, Brickmack, AvocatoBot, Rm1271, Altaïr, F=q(E+v^B), Snow Blizzard, Brad7777, Bhaskarandpm, Eduardofeld, GoShow, Dexbot, JoshyyP, Brandonsmacgregor, Reeceyboii, Frosty, Reatlas, I am One of Many, Eyesnore, Tentinator, Germeten, Nablacdy, Spyglasses, Freddyboi69, 20M030810, SpecialPiggy, Marizperoj, Peterfreed, Rigid hexagon, Jiteshkumar727464, Dyeith, Podayeruma, Oleaster, Layfi, BlueDecker, GeneralizationsAreBad, Pritam kumar Barik, KasparBot, Ramprakashsfc and Anonymous: 425

- **Electrostatic induction** *Source:* https://en.wikipedia.org/wiki/Electrostatic_induction?oldid=646442405 *Contributors:* Heron, Xanzzibar, Giftlite, Superborsuk, ELApro, LeeHunter, Bobo192, Bert Hickman, Alansohn, Miaow Miaow, Prashanthns, Jeff.hochberg@verizon.net, Bgwhite, YurikBot, Plhofmei, E2mb0t~enwiki, BOT-Superzerocool, SmackBot, Incnis Mrsi, Dethme0w, Dogears, Hgrobe, Chetvorno, Christian75, Epbr123, Jj137, JAnDbot, VoABot II, Dekimasu, Email4mobile, Teol~enwiki, VolkovBot, Shindo9Hikaru, ARUNKUMAR P.R, Enviroboy, Dustybunny, SieBot, Firefly322, WikipedianMarlith, Iohannes Animosus, Addbot, TStein, PV=nRT, Zorrobot, HerculeBot, PieterJanR, Luckas-bot, TaBOT-zerem, AnomieBOT, Xqbot, Robert9673, RedBot, Ezhuttukari, RjwilmsiBot, John of Reading, K6ka, JSquish, Ida Shaw, EWikist, Teapeat, Planetscared, ClueBot NG, Helpful Pixie Bot, Fylbecatulous, Fepeguinux, Lugia2453, Monkbot, Gfraz, Sania.qamar15 and Anonymous: 53

- **Coulomb's law** *Source:* https://en.wikipedia.org/wiki/Coulomb'{ }s_law?oldid=680669496 *Contributors:* Andre Engels, Peterlin~enwiki, Heron, Ryguasu, Patrick, Michael Hardy, Ixfd64, Delirium, Eric119, Looxix~enwiki, Ahoerstemeier, Darkwind, AugPi, Reddi, Choster, The Anomebot, E23~enwiki, David.Monniaux, Slawojarek, Robbot, Rorro, Prara, Kathy T, Giftlite, DocWatson42, Radius, Tom-, Yath, Karol Langner, Maximaximax, Icairns, Edsanville, Safety Cap, Aponar Kestrel, Discospinster, Vsmith, Laurascudder, Rbj, Foobaz, Pokrajac, Kjkolb, Sam Korn, Jumbuck, Mlessard, Rudchenko, Walkerma, Hdeasy, Gene Nygaard, Redvers, Falcorian, Oleg Alexandrov, MartinSpacek, StradivariusTV, Kzollman, Palica, Mendaliv, MZMcBride, Vegaswikian, The wub, Matt Deres, Pediadeep, Fresheneesz, Srleffler, Chobot, Keko~enwiki, YurikBot, Hairy Dude, Jimp, JabberWok, Wimt, NawlinWiki, Dtrebbien, RUL3R, E2mb0t~enwiki, Leotohill, Aristotle2600, Dna-webmaster, Lazcisco, Searchme, Kelovy, Enormousdude, Sbyrnes321, SmackBot, Jclerman, Mangoe, Ququ, InverseHypercube, KnowledgeOfSelf, Pavlovič, Eskimbot, Jab843, Pedrose, Yamaguchi⁇⁇, Pieter Kuiper, Khoshzaban.m, Complexica, AlexDitto, Metacomet, DHN-bot~enwiki, Mona, Fjmustak, Can't sleep, clown will eat me, Metal Militia, V1adis1av, Hoof Hearted, Drphilharmonic, Justin212k, Ligulembot, Sadi Carnot, DJIndica, Nmnogueira, HDarke, Cronholm144, Mbeychok, Bjankuloski06en~enwiki, Nonsuch, Tiki God, Enelson, Ckatz, Dicklyon, Mets501, Asdfqwe123, Robin Chen, Tawkerbot2, Nutster, Hedgefighter, Physic sox, WeggeBot, Danrah, Vttoth, Cydebot, Meno25, Christian75, FastLizard4, Editor at Large, Irigi, Headbomb, Jojan, Navigatr85, AntiVandalBot, Luna Santin, Seaphoto, Credema, Steelpillow, JAnDbot, Douga, MER-C, Acroterion, WRonG, WolfmanSF, =avallone, Causesobad, Dr. Morbius, Rettetast, Leyo, Qrystal, Hans Dunkelberg, JohnPritchard, SimpsonDG, Cpiral, M C Y 1008, New Hampshirite, NewEnglandYankee, DorganBot, Treisijs, Lseixas, Yecril, Sheliak, VolkovBot, Jeff G., AlnoktaBOT, Fabbe, Thurth, Oshwah, Kychot, Gwib, Hqb, Broadbot, Psyche825, Andy Dingley, Lian1238, Spinningspark, Ohiostandard, CMBJ, Kbrose, SieBot, Cwkmail, Foosnag, OKBot, Randomblue, ClueBot, RedGav, Avenged Eightfold, Mild Bill Hiccup, DragonBot, Kocher2006, Alexbot, Brews ohare, Jotterbot, PhySusie, Thingg, T68492, Gonfer, Addbot, Stariki, NjardarBot, ⁇⁇⁇⁇, Ehrenkater, Lightbot, Jan eissfeldt, ‏עוזי ו‎., Zorrobot, Snaily, Luckas-bot, Yobot, TaBOT-zerem, Julia W, THEN WHO WAS PHONE?, Backslash Forwardslash, AnomieBOT, Jim1138, Materialscientist, Joeyjoe10, Citation bot, Xqbot, TinucherianBot II, P99am, GrouchoBot, Vmcbride, Shadowjams, WaysToEscape, Samwb123, Constructive editor, FrescoBot, Lookang, MrMathematique, Citation bot 1, Elockid, 10metreh, RedBot, Toolnut, IVAN3MAN, Inbamkumar86, TobeBot, SchreyP, Lionaneesh, Popovvk, Unioneagle, Irfanyousufdar, EmausBot, Wik-

8.13.2   Images

8.13.3 Content license